Syllabus

Applied Mathematics – I

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RATIONALE

Contents of this course provide fundamental base for understanding engineering problems and their solution algorithms. Contents of this course will enable students to use basic tools like logarithm, binomial theorem, matrices, t-ratios and co-ordinates for solving complex engineering problems with exact solutions in a way which involve less computational task. By understanding the logarithm, they will be able to make long calculations in short time and it is also a pre-requisite for understanding Calculus.

COURSE OUTCOMES

After undergoing this subject, the students will be able to:

CO1: Understand the geometric shapes used in engineering problems by Co-ordinate Geometry and Trigonometry.

CO2: Formulate engineering problems into mathematical formats with the use matrices, co-ordinate geometry and trigonometry

CO3: Calculate the approximate value of roots of certain expressions in engineering problems by application of binomial theorem.

CO4: Explore the idea of location, graph, and linear relationships between two variables.

CO5: Learn about basic fundamentals about MATLAB/ SciLab and mathematical calculation with MATLAB/ SciLab software.

DETAILED CONTENTS

UNIT I

Algebra

1.1 Complex Numbers: definition of complex number, real and imaginary parts of a complex number, Polar and Cartesian Form and their inter conversion, Conjugate of a complex number, modulus and amplitude, addition subtraction, multiplication and division of complex numb

1.2 Logarithms and its basic properties

UNIT II

Binomial Theorem, Determinants and Matrices

2.1 Meaning of npr&ncr (mathematical expression). Binomial theorem (without proof) for positive integral index (expansion and general form); binomial theorem for any index (expansion up to 3 terms - without proof), first binomial approximation with application to engineering problems.

2.2 Determinants and Matrices – Evaluation of determinants (upto 2ndorder), solution of equations (upto 2 unknowns) by Crammer’s rule, definition of Matrices and its types, addition, subtraction and multiplication of matrices (upto 2nd order).

UNIT III

Trigonometry

3.1 Concept of angle, measurement of angle in degrees, grades, radians and their conversions.

3.2 T-Ratios of Allied angles (without proof), Sum, Difference formulae and their applications (without proof). Product formulae (Transformation of product to sum, difference and vice versa

3.3 Applications of Trigonometric terms in engineering problems such as to find an angle of elevation, height, distance etc.

UNIT IV

Co-ordinate Geometry

4.1 Cartesian and Polar co-ordinates (two dimensional), Distance between two points, mid-point, centroid of vertices of a triangle.

4.2 Slope of a line, equation of straight line in various standards forms (without proof); (slope intercept form, intercept form, one-point form, two-point form, symmetric form, normal form, general form), intersection of two straight lines, concurrency of lines, angle between straight lines, parallel and perpendicular lines, perpendicular distance formula, conversion of general form of equation to the various forms.

UNIT V

Geometry of Circle and Software

Circle

5.1 General equation of a circle and its characteristics. To find the equation of a circle, given:

I. Centre and radius

II. Three points lying on it

III. Coordinates of end points of a diameter

Software

5.2 **MATLAB Or SciLab software –** Theoretical Introduction, MATLAB or Scilab as Simple Calculator (Addition and subtraction of values –Trigonometric and Inverse Trigonometric functions) – General Practice

UNIT I
ALGEBRA

Learning Objectives

- To understand the need for extending the set of real numbers to the set of complex numbers.
- To learn the use of algebraic operations on complex numbers.
- To use the laws of logarithms to simplify and expand logarithmic numerical and algebraic expressions.

1.1 COMPLEX NUMBERS

Number System: We know the number system as

(1) Natural numbers, N = {1, 2, 3, …, }

(2) Whole Numbers, W = {0, 1, 2, 3…}

(3) Integers, Z = {…,–3, –2, –1, 0, 1, 2, 3, …}

(4) Rational numbers, Q = $\left\{\frac{p}{q}, p, q \in Z, q \neq 0\right\}$

(5) Irrational numbers – The numbers whose decimal representation is non–terminating and non repeating

e.g. $\sqrt{2}, \sqrt{3}, \pi$ etc.

(6) Real numbers (R) = (Rational numbers + Irrational numbers)

Let us take an quadratic equation; $x^2 + 7x + 12 = 0$ which has real root –4 and –3, *i.e.* solution of $x^2 + 7x + 12 = 0$ is x = –4 and x = –3 which are both real numbers.

But for the quadratic equation of the form $4x^2 - 4x + 5 = 0$, no real value of x satisfies the equation. For the solution of the equations of such types the idea of complex numbers is introduced.

Imaginary Numbers or Complex numbers:

Solution of quadratic equation $x^2 + 4 = 0$

or $x^2 = -4$

$$x = \sqrt{-4} = \sqrt{-1 \times 4} = \sqrt{-1} \times \sqrt{4} = i \times 2$$

where $i = \sqrt{-1}$ is an imaginary number called as iota.

The square root of a negative number is always imaginary number.

e.g. $\sqrt{-4}, \sqrt{-16}, \sqrt{-25}, \sqrt{-\frac{9}{16}}$ etc. are all imaginary numbers.

i.e. $\sqrt{-4} = 2i,\ \sqrt{-16} = 4i,\ \sqrt{-25} = 5i$

and $\sqrt{-\frac{9}{16}} = \frac{3}{4}i$

Thus the solution of quadratic equation $4x^2 - 4x + 5 = 0$ are :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2 \times 4}$$

$$x = \frac{4 \pm \sqrt{16 - 80}}{8}$$

$$x = \frac{4 \pm \sqrt{-64}}{8} = \frac{4 \pm 8i}{8} = \frac{4 + 8i}{8}, \frac{4 - 8i}{8}$$

$$\Rightarrow \quad \frac{1 + 2i}{2}, \frac{1 - 2i}{2}$$

Thus the given quadratic equation has complex roots.

Powers of iota (*i*)

(i) $i = \sqrt{-1}$

(ii) $i^2 = 1$

(iii) $i^3 = i^2.i = (-1)i = -i$

(iv) $i^4 = (i^2)^2 = (-1)^2 = 1$

(v) $i^5 = i^4.i = i$

(vi) $i^{24} = (i^4)^6 = (1)^6 = 1$

(vii) $i^{25} = (i^4)^6 i = 1 \times i = i$

(viii) $i^{4n+1} = (i^4)^n i = 1 \times i = i$

Real and Imaginary part of Complex number : A number of the form x + iy, where x and y are real numbers and i is an imaginary number with property $i^2 = -1$ i.e. $i = \sqrt{-1}$ is called a complex number. The complex number is denoted by Z.

$$\therefore \qquad Z = x + i\,y$$

Here x is called real part of Z and denoted by Re(Z) and y is called imaginary part of Z, it is denoted by Im(Z).

Thus complex number Z = x + iy can be represented as

$$Z = \text{real part} + \boldsymbol{i}\,(\text{Imaginary part})$$

Examples of complex numbers are :

(i) Z = 2 + i, where Re(Z) = 2, Im(Z) = 1

(ii) Z = 4 − 7i, where Re(Z) = 4, Im(Z) = −7

(iii) $Z = -\frac{2}{3} + \frac{5}{3}i$, where $Re(Z) = -\frac{2}{3}$, $Im(Z) = \frac{5}{3}$

(iv) Z = 4 + 0i , where Re(Z) = 4, Im(Z) = 0

Here Im(Z) = 0, such type of complex number is known as purely real

(v) Z = 0 + 3i, where Re(Z) = 0, Im(Z) = 3

Hence Re(Z) = 0, hence given complex number is called purely imaginary number.

Properties of Complex Numbers:

(i) Equality of two complex number: Let $Z_1 = x_1 + iy_1$ and $Z_2 = x_2 + iy_2$ are two complex numbers.

If $Z_1 = Z_2 \quad \Rightarrow \quad x_1 + iy_1 = x_2 + iy_2$

Then $x_1 = x_2$ and $y_1 = y_2$

i.e. their real and imaginary part are separately equal.

(ii) if $x + iy = 0$, then $x = 0$ and $y = 0$ *i.e.* if a complex number is zero then its real part and imaginary part both are zero.

Example 1. Solve the equation $2x + (3x + y)i = 4 + 10i$.

Sol. Using property (i)

$$2x = 4 \text{ and } 3x + y = 10$$

$\Rightarrow$ $x = 2$, Putting value of $x = 2$ in

$$3x + y = 10, \text{ we get } 6 + y = 10 \Rightarrow y = 10 - 6 = 4$$

Hence $x = 2$, $y = 4$

Example 2. Find x & y if $\frac{1}{x} + \frac{1}{y}i = 2 + 3i$.

Sol. Equating real and imaginary part, we get

$$\frac{1}{x} = 2 \quad \Rightarrow \quad x = \frac{1}{2}$$

and $$\frac{1}{y} = 3 \quad \Rightarrow \quad y = \frac{1}{3}$$

Conjugate of a Complex Number

If $z = x + iy$ is a complex number than conjugate of Z is $x - iy$. It is denoted by $\bar{z}$.

$$\therefore \quad \bar{z} = x - iy$$

e.g. if $z = 2 + 3i$, then conjugate of z is $\bar{z} = 2 - 3i$.

Properties of conjugate of complex number

(i) $\overline{(\bar{z})} = z$

(ii) $(\overline{z_1 + z_2}) = \bar{z}_1 + \bar{z}_2$

(iii) $\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$

x–axis is called real axis
y–axis is called imaginary.

Example 3. Write conjugate of $z = 4i^3 + 3i^2 + 5i$.

Sol. Given that

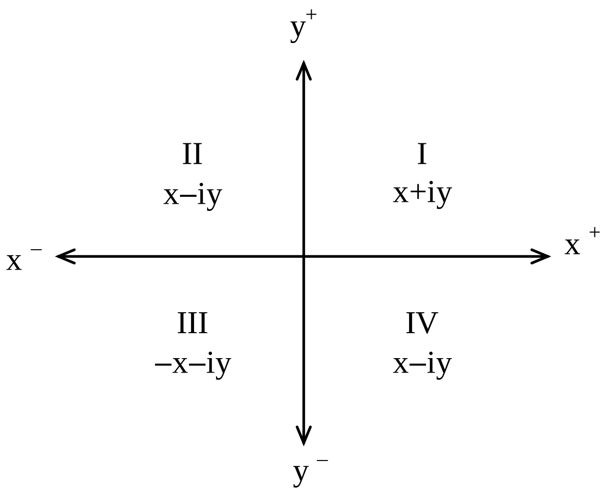
$$z = 4i^3 + 3i^2 + 5i$$

$$= 4i^2 i + 3(-1) + 5i$$

$$z = -4i - 3 + 5i = -3 + i$$

Conjugate of z is $-3 - i$.

Sign of complex number in quadrant system Fig 1.1



(Fig). 1.1

Polar and Cartesian form of a complex number.

Let $z = x + iy$ is a complex number in Cartesian form represented by a Point P(x, y) in Argand plane (XY–plane) as shown in Fig. 1.2

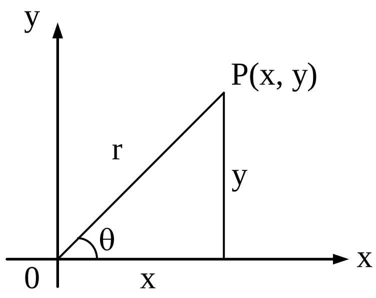


Figure .1.2

Then $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore$ $y = r\sin\theta$ …(1) and $x = r\cos\theta$ (2)

The complex number $Z = x + iy = r\cos\theta + i\, r\sin\theta = r[\cos\theta + i\sin\theta]$ is a polar form of given complex number. Squaring and adding (1) and (2), we get

$$x^2 + y^2 = r^2[\cos^2\theta + \sin^2\theta]$$

$$= r^2[1]$$

$\Rightarrow$ $r = \sqrt{x^2 + y^2}$ $\Rightarrow$ Modulus of $z = x + iy$ *i.e.* r is called the modulus of z

Dividing (1) by (2), we get

$$\tan\theta = \frac{y}{x}$$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\Rightarrow$ The argument or amplitude of z. i.e. θ is called the argument or amplitude of z.

$\therefore$ Polar form of a complex number z is

$$Z = x + iy = r[\cos\theta + i\sin\theta]$$

Thus $Z = x + iy$ is Cartesian form of Z.

and $Z = r[\cos\theta + i\sin\theta]$ is Polar form of Z

Where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Conversion from Cartesian Form to Polar Form

Let x + iy be a complex number in Cartesian form, then we have to convert into polar form

$$x + iy \xrightarrow{\text{Change}} r(\cos\theta + i\sin\theta)$$

Putting the value of r and θ, we get required form

i.e. $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

∴ $x + iy = r(\cos\theta + i\sin\theta)$

$$= \sqrt{x^2 + y^2}\left[\cos(\tan^{-1}\left(\frac{y}{x}\right)\right] + i\sin\left[\tan^{-1}\left(\frac{y}{x}\right)\right]$$

Example 4. Express $1 + \sqrt{3}i$ into polar form.

Sol. Let z = $1 + \sqrt{3}i$, here x = 1, $y = \sqrt{3}$

Polar form of complex number is

$$z = r[\cos\theta + i\sin\theta]$$

We know, $r = \sqrt{x^2 + y^2} = \sqrt{1+3} = \sqrt{4} = 2$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1} = \sqrt{3} = \tan\frac{\pi}{3}$$

⇒ $\theta = \frac{\pi}{3}$

∴ Required polar form is

$$z = r[\cos\theta + i\sin\theta] = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$$

Example 5. Convert 1 – i into polar form.

Sol. Let Z = 1 – i, then x = 1, y = –1

$$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1,$$ z lies in IV quadrant

$$\tan\theta = -1 = \tan\left(2\pi - \frac{\pi}{4}\right) = \tan\frac{7\pi}{4}$$

$$\theta = \frac{7\pi}{4}$$

∴ Required form of Z is

$$Z = r(\cos\theta + i\sin\theta)$$

$$Z = \sqrt{2}[\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}]$$

Conversion from Polar form to Cartesian Form

Let Z = r(cos θ + i sin θ) be the polar form and x + iy be its rectangular form.

Put x = r cos θ, y = r sin θ to get required form.

Example 6. Convert 4(cos 300° + i sin 300°) into Cartesian from.

Sol. $4(\cos 300° + i\sin 300°) \xrightarrow{\text{change}} x + iy$

Put $x = 4\cos 300^o = 4\cos(360^o - 60^o) = 4\cos 60^o = 4 \times \frac{1}{2} = 2$

$$y = 4\sin 300° = 4\sin(360° - 60°) = -4\sin 60° = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

∴ Required Cartesian form is

$$x + iy = 2 - 2\sqrt{3}i$$

Modulus and Amplitude of a Complex Number

If z = x + iy is a complex number, then

a) $r = \sqrt{x^2 + y^2} = \sqrt{(\text{real part})^2 + (\text{Imaginary part})^2}$ is known as Modulus of z.

it is denotes by | z |.

Thus Modulus of z = | z | = r = $\sqrt{x^2 + y^2}$

(b) Amplitude of z

$$\tan\theta = \frac{y}{x} \text{ i.e., } \tan\theta = \frac{im(z)}{re(z)}$$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ is known as Amplitude or argument of z.

Example 7. Find the Modulus and Amplitude of a + ib.

Sol. Here z = a + ib

(i) Modulus, |z| = $\sqrt{a^2 + b^2}$

(ii) Amplitude, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example 8. Find Modulus and amplitude of the complex number 1 + i.

Sol. Here z = 1 + I i.e. x = 1, y = 1

(i) Modulus | z | = $\sqrt{x^2 + y^2} = \sqrt{1 + 1} = \sqrt{2}$

(ii) Amplitude : $\tan\theta = \frac{y}{x} = \frac{1}{1}$

$$\tan\theta = 1 = \tan\frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}$$

Example 9. Find the Modulus and Amplitude of the complex number −1 + i.

Sol. Let $z = -1 + i$

Compare it with z = x + iy, we get

$$x = -1, y = 1$$

(1) Modulus of z = $|z| = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

(2) Amplitude of z $\Rightarrow \tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$

Complex number −1 +i lie in II quadrant.

θ also lie in IInd quadrant.

$$\tan\theta = -1 = \tan(180° - 45°) = \tan 135°$$

$\therefore$ $\theta = 135°$ or $\theta = \frac{3\pi}{4}$.

Example 10. Find modulus of each of the complex numbers 6 + 7i and 1 + 10i.

Sol. Let $z_1 = 6 + 7i$, $z_2 = 1 + 10i$

Then Modulus of $z_1 = |z_1| = \sqrt{6^2 + 7^2} = \sqrt{36 + 49} = \sqrt{85}$

Modulus of $z_2 = |z_2| = \sqrt{(1)^2 + (10)^2} = \sqrt{1 + 100} = \sqrt{101}$

Example 11. Find modulus and amplitude of the complex number $-1 + \sqrt{3}i$.

Solution : Let z = $-1 + \sqrt{3}i$

then $x = -1$, $y = \sqrt{3}$

(1) Modulus of z = | z | = $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=2$

(2) Amplitude of z ⇒ $\tan\theta = \frac{y}{x}=\frac{\sqrt{3}}{-1}=-\sqrt{3}$

Complex number lie in IInd quadrant.

∴ $$\tan\theta=-\sqrt{3}=\tan\left(\pi-\frac{\pi}{3}\right)=\tan\frac{2\pi}{3}$$

$$\theta=\frac{2\pi}{3}.$$

Addition, Subtraction, Multiplication and Division of Complex Numbers :

(i) Addition of Complex Numbers : Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex numbers.

Then $$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$

$$= (x_1 + x_2) + i(y_1 + y_2)$$

i.e add real parts and Im. parts separately.

Note : Addition of two complex numbers is also a complex number.

Example 12. Let $z_1 = 7 + 3i$, $z_2 = 9 - i$ then

$$z_1 + z_2 = (7 + 3i) + (9 - i)$$

$$= (7 + 9) + i(3 - 1) = 16 + 2i$$

Note $$z_1 + z_2 = z_2 + z_1$$

(ii) Subtraction of Complex Numbers :

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex number.

then $$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2)$$

$$= (x_1 - x_2) + i(y_1 - y_2)$$

Note – Difference of two complex numbers is also complex number.

Example 13. Let $z_1 = 2 + 4i$, $z_2 = 7 + 5i$

then $z_1 - z_2 = (2 + 4i) - (7 + 5i)$

$= (2 - 7) + i(4 - 5)$

$= -5 - i$

Note $z_1 - z_2 \neq z_2 - z_1$

(iii) Multiplication of Complex Numbers

(a) In Cartesian form

Let $z_1 = x + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then

$$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1x_2 + ix_1y_2 + ix_2y_1 + i^2y_1y_2 \quad (i^2 = -1)$$

$$z_1z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

(b) In polar form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_2)$ and

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ are two complex number in polar form.

Then $z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

Note : Multiplication of two complex number is also a complex number.

(iv) Division of two complex numbers

(a) In Cartesian form

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2}$$

$$= \frac{x_1x_2 - ix_1y_2 + ix_2y_2 - i^2y_1y_2}{x_2^2 - i^2y_2^2}$$

$$= \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2} \qquad (i^2 = -1)$$

$\therefore \qquad \dfrac{z_1}{z_2} = \left(\dfrac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}\right) + i\left(\dfrac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}\right)$

Note : Division of two complex numbers is also a complex number.

(b) In Polar form

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + \sin\theta_1)}{r_2(\cos\theta_2 + \sin\theta_2)}$$

$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Example 14. if $z_1 = -2 + 4i$ and $z_2 = 1 - 3i$, then find z_1z_2.

Sol. $z_1z_2 = (-2 + 4i)(1 - 3i)$

$= -2 + 6i + 4i - 12i^2$ $\qquad (i^2 = -1)$

$= -2 + 10i + 12$

$= 10 + 10i$

Example 15. if $z_1 = 5(\cos 30° + i \sin 30°)$ and $z_2 = 2(\cos 30° + i \sin 30°)$. Find z_1z_2.

Sol. $z_1z_2 = 5\times 2[\cos(30+30) + i\sin(30+30)]$

$= 10[\cos 60° + i \sin 60°]$

$= 10\left[\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right]$

$= 5 + 5\sqrt{3}i$

Example 16. If $z_1 = 2 - i$, $z_2 = 2 + i$, then

$$\frac{z_1}{z_2} = \frac{2-i}{2+i} = \frac{2-i}{2+i}\times\frac{2-i}{2-i}$$

$$= \frac{(2-i)^2}{(2)^2 - (i)^2} = \frac{4 + i^2 - 4i}{4+1}$$

$$= \frac{3-4i}{5} = \frac{3}{5} - \frac{4i}{5}$$

Example 17. If $z_1 = 5 + 7i$, $z_2 = 9 - 3i$, find $\dfrac{z_1}{z_2}$

Sol. $$\frac{z_1}{z_2} = \frac{5+7i}{9-3i} = \frac{5+7i}{9-3i} \times \frac{9+3i}{9+3i}$$

$$= \frac{45+15i+63i+21i^2}{(9)^2-(3i)^2}$$

$$= \frac{(45-21)+i(15+63)}{81+9} = \frac{24+78i}{90}$$

$$= \frac{24}{90} + \frac{78}{90}i$$

Example 18. If $z_1 = 50[\cos 50° + i \sin 50°]$ and $z_2 = 10[\cos(-10°) + i \sin(-10°)]$

then $$\frac{z_1}{z_2} = \frac{50}{10}[\cos(50-(-10)) + i\sin[50-(-10)]]$$

$$= 5[\cos 60° + i \sin 60°]$$

$$= 5\left[\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= \frac{5}{2} + \frac{5\sqrt{3}}{2}i$$

Example 19. Find modulus of $z = 4i^3 + 3i^2 + 5i$

given $z = 4(-i) - 3 + 5i$

$z = -4i - 3 + 5i$

$z = -3 + i$

$$|z| = \sqrt{(-3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

(V) Multiplicative Inverse of a Complex Number

Let $z = x + iy$ be a complex number then multiplicative inverse of z is $\frac{1}{z}$

i.e. $$\frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$$

$$= \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

Example 20. Find the multiplicative inverse (MI) of 1 – 2i.

Sol. MI of z is given by $\frac{1}{z} = \frac{1}{1-2i}$

$$= \frac{1}{1-2i} \times \frac{1+2i}{1+2i} = \frac{1+2i}{1+4}$$

$$= \frac{1}{5} + \frac{2}{5}i$$

Example 21. Find the value of x and y if 3x + (2x – y)i = 6 – 3i.

Sol. Equating real and imaginary part, we get

$$3x = 6 \quad \Rightarrow \quad x = 2$$

and $2x - y = -3$

$$2(2) - y = -3 \quad \Rightarrow \quad 4 - y = -3$$

$$-y = -7 \quad \Rightarrow \quad y = 7$$

Example 22. Express in complex form $\frac{2-i}{(1-2i)^2}$.

Sol. $\frac{2-i}{(1-2i)^2} = \frac{2-i}{1+4i^2-4i} = \frac{2-i}{-3-4i}$

$$= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i} = \frac{-6+8i+3i-4i^2}{9+16}$$

$$= \frac{-2+11i}{25} = \frac{-2}{25} + \frac{11}{25}i$$ which is required x + iy form

Example 23. Simplify $\left(\frac{1}{2}+2i\right)^3$

Sol. $\left(\frac{1}{2}+2i\right)^3 = \left(\frac{1}{2}\right)^3 + (2i)^3 + 3\left(\frac{1}{2}\right)(2i)\left[\frac{1}{2}+2i\right]$

$$= \frac{1}{8} + 8i^3 + 3i\left(\frac{1}{2}+2i\right)$$

$$= \frac{1}{8} - 8i + \frac{3}{2}i + 6i^2$$

$$= \frac{1}{8} - 8i + \frac{3}{2}i - 6 = \frac{-47}{8} - \frac{13}{2}i$$

Example 24. Find multiplicative inverse of z = $(6 + 5i)^2$.

Sol. $z = (6 + 5i)^2 = 36 + 25i^2 + 60i$

$= 36 - 25 + 60i$

$= 11 + 60i$

M.I. of $z = \frac{1}{z} = \frac{1}{11+60i} = \frac{1}{11+60i} \times \frac{11-60i}{11-60i}$

$= \frac{11-60i}{121+3600} = \frac{11-60i}{3721}$

$= \frac{11}{3721} - \frac{60}{3721}i$

Example 25. Express $\frac{1}{3+i} - \frac{1}{3-i}$ in x + iy form

Sol. $\frac{1}{3+i} - \frac{1}{3-i} = \frac{3-i-3-i}{(3+i)(3-i)} = \frac{-2i}{9+1}$

$= \frac{0-2i}{10}$ $= 0 - \frac{1}{5}i$ is the required form.

Example 26. Express $\frac{(3+i)(4-i)}{5+i}$ in the form a + ib.

Sol. $\frac{(3+i)(4-i)}{5+i} = \frac{12-3i+4i-i^2}{5+i} = \frac{13+i}{5+i}$

$\frac{13+i}{5+i} \times \frac{5-i}{5-i} = \frac{65-13i+5i-i^2}{25+1} = \frac{66-8i}{26}$

$\frac{66}{26} - \frac{8}{26}i$ $= \frac{33}{13} - \frac{4}{13}i$ is required a + ib form.

Example 27. Find Modulus and amplitude of $\frac{1+i}{1-i}$.

Sol. Let $z = \frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1^2+i^2} = \frac{(1+i)^2}{1+1}$

$$z = \frac{1+i^2+2i}{2} = \frac{1-1+2i}{2} = \frac{2i}{2} = i$$

$z = i = 0 + i$

(a) Modulus $|z| = \sqrt{(0)^2+(1)^2} = 1$

(b) Amplitude $= \tan\theta = \frac{y}{x} = \frac{1}{0} = \infty = \tan\frac{\pi}{2}$

$$\theta = \frac{\pi}{2}.$$

Example 28. Simplify $1+i^{100}+i^{10}+i^{50}$.

Sol. $1+i^{100}+i^{10}+i^{50} = 1+(i^4)^{25}+i^8.i^2+i^{48}.i^2$

$= 1+(1)^{25}+(i^4)^2 i^2+(i^4)^{12} i^2$

$= 1 + 1 + (1).i^2 + (1)\, i^2$

$= 1 + 1 - 1 - 1 = 0$

EXERCISE –I

Questions on complex numbers:

1. If (x + iy) (2 – 3i) = 4 – i, find x and y.

2. If $(1+i)(x+iy) = 2-5i$, find x and y.

3. Find real value of x and y if (1 – i)x + (1 + i)y = 1 – 3i.

4. Evaluate (i) i^{25} (ii) i^{19}

5. Find modulus and Amplitude of following

   (i) $1+\sqrt{3}i$

(ii) $1 + i$

(iii) $4\sqrt{3} + 4i$

(iv) $z = \frac{3+2i}{4-5i}$

6. Add 2 + 3i and 5 – 6i

7. Subtract 7 – 5i from 2+4*i*

8. Simplify (5 + 5i) (4 – 3i)

9. If $z_1 = 1 + 3i$, $z_2 = 2 + i$, find z_1z_2

10. Write $\frac{3+4i}{2-3i}$ in x + iy form

11. Simplify $\frac{4-7i}{3-2i}$

12. Find multiplicative inverse of 3 + 4i

13. Write conjugate of –3 + 2i

14. Write conjugate of $\frac{3+2i}{4-5i}$

15. Express $\frac{(2+3i)^2}{1-i}$ is x + iy form

16. $z_1 = 2(\cos 60° + i \sin 60°)$, $z_2 = 4(\cos 15° + i \sin 15°)$, find $\frac{z_1}{z_2}$.

17. Write $\left(\frac{3}{2} + \frac{\sqrt{5}}{2} i\right)^2$ in x + iy form

18. Find modulus and conjugate of $\frac{2+7i}{3+2i}$

19. Express $\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$ into x + iy form

20. Express $\frac{2+4i}{2-3i}$ in complex form x + *i*y

21. Write $\frac{(1-i)(2-i)(3-i)}{1+i}$ in x + iy form

22. Find the conjugate of $(3 - 7i)^2$

23. Find the amplitude of z if z = $\frac{-1-\sqrt{3}\,i}{2}$

24. The value of $\frac{i+i^2+i^3+i^4+i^5}{i+1}$ is

(a) $\frac{1-i}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1+i}{2}$

25. If $(x+\sqrt{y})(p+\sqrt{q}) = x^2 + y^2$ then

(a) p = x, q = y (b) $p = x^2$, $q = y^2$ (c) p = y, q = x (d) None of these

26. $(3a^{-2} + 2b^{-1})^2 =$

(a) $9a^{-4} + 4b^{-2} + 12a^{-2}b^{-1}$ (b) $9a^0 + 4b^0 + 12a^{-2}b^{-1}$

(c) $9a^{-2} + 4b^{-4} + 12a^{-2}b^{-1}$ (d) None of these

27. The value of $\sqrt{(-1)(-1)}$ is

(a) 1 (b) -1 (c) *i* (d) -*i*

28. The roots of the equation, $ax^2 + 2bx + c = 0$ are

(a) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $\frac{-b \pm \sqrt{4b^2 - 4ac}}{2a}$

(c) $\frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a}$ (d) None of these

29. The factorization of $x^2 - 5x + 6 = 0$ is

(a) (x+2) (x-3) (b) (x+2) (x+3)

(c) (x-2) (x-3) (d) (x-2) (x+3)

30. In a proper fraction;

(a) The degree of numeration is equal to the degree of denominator

(b) The degree of numeration is less than the degree of denominator

(c) The degree of numeration is more than the degree of denominator

(d) None of these

31. The partial fractions of $\frac{x+3}{x^2+x}$ are

(a) $\frac{2}{x}-\frac{3}{x+1}$ (b) $-\frac{2}{x}+\frac{3}{x+1}$

(c) $\frac{3}{x}-\frac{2}{x+1}$ (d) $\frac{3}{x}+\frac{2}{x+1}$

32. The multiplicative inverse of 1 + *i* is

(a) $\frac{1}{2}(1-i)$ (b) $\frac{1}{2}(1+i)$

(c) (1 - *i*) (d) None of these

33. The real & imaginary parts of $\frac{(2+3i)^2}{1-i}$ are

(a) $\frac{22}{5}$ & $\frac{19}{5}$ (b) $\frac{22}{5}$ & $-\frac{19}{5}$ (c) $-\frac{22}{5}$ & $\frac{19}{5}$ (d) None of these

34. Amplitude of $\frac{-1-\sqrt{3}i}{2}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

35. Conjugate of $(2 + i)^2$ is

(a) -3 + 4*i* (b) 3 - 4*i* (c) -3 - 4*i* (d) None of these

ANSWERS

1. $x=\frac{11}{13},\ y=\frac{10}{13}$ 2. $x=\frac{-3}{2}, y=-\frac{7}{2}$ 3. x = 2, y = –1

4. (i) i (ii) – I 5. (i) $r = 2,\ \theta=\frac{\pi}{3}$ (ii) $r=\sqrt{2}, \theta=\frac{\pi}{4}$ (iii) r = 8, $\theta=\frac{\pi}{6}$

(iv) $|z| = \frac{\sqrt{533}}{41}$, $\boldsymbol{\theta} = \tan^{-1}\left(\frac{23}{2}\right)$ 6. 7 – 3i 7. –5 + 9i

8. [35 + 5i]

9. −1 + 7i

10. $\frac{-6}{13}+\frac{17}{13}i$

11. $10\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]$

12. $\left(\frac{3}{25}\right)+\left(\frac{-4}{25}i\right)$

13. −3 − 2i

14. $\frac{2}{41}-\frac{23}{41}i$

15. $-\frac{17}{2}+\frac{7}{2}i$

16. $-\frac{1}{2}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right]$

17. $1+\frac{3\sqrt{5}}{2}i$

18. $|z|=\frac{\sqrt{689}}{13}$ $\bar{z}=\frac{10}{13},\frac{17}{13}i$

19. $\frac{-8}{29}$

20. $-\frac{6}{13}+\frac{17}{13}i$

21. 5 + 5i

22. −40 + 42i

23. $\frac{4\pi}{3}$

24. (d)

25. (d)

26. (a)

27. (a)

28. (c)

29. (c)

30. (b)

31. (c)

32. (a)

33. (c)

34. (d)

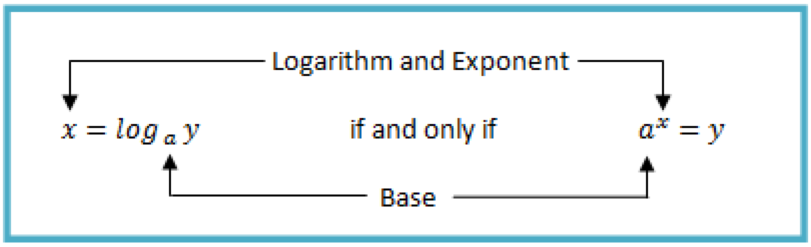
35. (b)

1.2 LOGARITHM

Definition: If y and a are positive real numbers($a \neq 1$), then $x = log_a y$ if and only if $a^x = y$.

The notation $log_a y$ is read as "log to the base a of y". In the equation $x = log_a y$, x is known as the **logarithm**, a is the **base** and y is the **argument**.

Note: 1. The above definition indicates that a logarithm is an exponent.



2. Logarithm of a number may be negative but the argument of logarithm must be positive. The base must also be positive and not equal to 1.

3.

Logarithmic Form	Exponential Form
$x = log_a y$	$a^x = y$

4. Logarithm of zero doesn't exist.

5. Logarithms of negative real numbers are not defined in the system of real numbers.

6. Log to the base "10" is called Common Logarithm and Log to the base "e" is called Natural Logarithm.($e = 2.7182818284 \ldots$)

7. If base of logarithm is not given, we'll consider it Natural Logarithm.

Some examples of logarithmic form and their corresponding exponential form:

S. No.	Logarithmic form	Exponential form
1	$5 = log_2 32$	$2^5 = 32$
2	$4 = log_3 81$	$3^4 = 81$
3	$3 = log_5 125$	$5^3 = 125$
4	$4 = log_{10} 10000$	$10^4 = 10000$
5	$-2 = log_7 \left(\frac{1}{49}\right)$	$7^{-2} = \frac{1}{49}$
6	$0 = log_e 1$	$e^0 = 1$

Why do we study logarithms: Sometimes multiplication, subtraction and exponentiation become so lengthy and tedious to solve. Logarithms covert the problems of multiplication into addition, division into subtraction and exponentiation into multiplication, which are easy to solve.

Properties of Logarithms: If a, b and c are positive real numbers, $a \neq 1$ and n is any real number, then

1. **Product property:** $\boldsymbol{log_a(b.c) = log_a b + log_a c}$

   For example: $log_{10}(187) = log_{10}(11 \times 17) = log_{10}11 + log_{10}17$

2. **Quotient property:** $\boldsymbol{log_a\left(\frac{b}{c}\right) = log_a b - log_a c}$

   For example: $log_7\left(\frac{51}{7}\right) = log_7 51 - log_7 7$

3. **Power property:** $\boldsymbol{log_a b^n = n. log_a b}$

   For example: $log_{10}(10000) = log_{10}(10^4) = 4. log_{10}10$

4. **One to One property:** $\boldsymbol{log_a b = log_a c}$ **if and only if** $\boldsymbol{b = c}$**.**

   For example: If $log_{10}(a) = log_{10}(15)$ then $a = 15$.

5. $\boldsymbol{log_a 1 = 0}$

   For example: $log_{10}(1) = 0$, $log_2(1) = 0$, $log_e(1) = 0$ etc.

6. $\boldsymbol{log_a a = 1}$

   For example: $log_{10}(10) = 1$, $log_e(e) = 1$ etc.

7. $\boldsymbol{log_a a^n = n}$

   For example: $log_{10}10^4 = 4$

8. $\boldsymbol{a^{log_a(n)} = n}$**, where** $\boldsymbol{n} > 0$

   For example: $2^{log_2(8)} = 8$

9. **Change of base property:** $\boldsymbol{log_a b = \frac{log(b)}{log(a)} = \frac{log_c(b)}{log_c(a)}}$ **provided that** $\boldsymbol{c \neq 1}$**.**

For example: $log_2(3) = \frac{log_{10}(3)}{log_{10}(2)}$ (Here we changed the base to 10)

Some solved problems:

Example 29. Convert the following exponential forms into logarithmic forms:

(i) $9^3 = 729$ **(ii)** $7^5 = 16807$ **(iii)** $2^{10} = 1024$

(iv) $10^{-3} = 0.001$ **(v)** $4^{-2} = 0.0625$ **(vi)** $5^{-4} = 0.0016$

(vii) $10^0 = 1$ **(viii)** $8^0 = 1$

Ans. (i) Given that $9^3 = 729$

$\Rightarrow \quad log_9 729 = 3 \qquad (by\ definition)$

which is required logarithmic form.

OR

Given that $9^3 = 729$

Taking logarithm on both sides, we get

$log 9^3 = log 729$

$\Rightarrow \quad 3\, log 9 = log 729 \qquad (used\ power\ property)$

$\Rightarrow \quad 3 = \frac{log 729}{log 9}$

$\Rightarrow \quad 3 = log_9 729 \qquad \left(used\ log_a b = \frac{log(b)}{log(a)}\right)$

which is required logarithmic form.

(ii) Given that $7^5 = 16807$

$\Rightarrow \quad log_7 16807 = 5 \qquad (by\ definition)$

which is required logarithmic form.

OR

Given that $7^5 = 16807$

Taking logarithm on both sides, we get

$log 7^5 = log 16807$

$\Rightarrow \quad 5\, log 7 = log 16807 \qquad (used\ power\ property)$

$\Rightarrow \quad 5 = \frac{log 16807}{log 7}$

$\Rightarrow \quad 5 = log_7 16807 \qquad \left(used\ log_a b = \frac{log(b)}{log(a)}\right)$

which is required logarithmic form.

(iii) Given that $2^{10} = 1024$

$\Rightarrow \quad log_2 1024 = 10$

which is required logarithmic form.

(iv) Given that $10^{-3} = 0.001$

$\Rightarrow \quad log_{10} 0.001 = -3$

which is required logarithmic form.

(v) Given that $4^{-2} = 0.0625$

$\Rightarrow \quad log_4 0.0625 = -2$

which is required logarithmic form.

(vi) Given that $5^{-4} = 0.0016$

$\Rightarrow \quad log_5 0.0016 = -4$

which is required logarithmic form.

(vii) Given that $10^0 = 1$

$\Rightarrow \quad log_{10} 1 = 0$

which is required logarithmic form.

(viii) Given that $8^0 = 1$

$\Rightarrow \quad log_8 1 = 0$

which is required logarithmic form.

Example 30. Convert the following logarithmic forms into exponential forms:

(i) $log_\pi 1 = 0$ **(ii)** $log_2 2048 = 11$ **(iii)** $log_4 \left(\frac{1}{64}\right) = -3$

(iv) $log_3 243 = 5$ **(v)** $log_{10} 0.01 = -2$

Ans. (i) Given that $log_\pi 1 = 0$

$\Rightarrow \quad \pi^0 = 1$

which is required exponential form.

(ii) Given that $log_2 2048 = 11$

$\Rightarrow \quad 2^{11} = 2048$

which is required exponential form.

(iii) Given that $log_4 \left(\frac{1}{64}\right) = -3$

$\Rightarrow \quad 4^{-3} = \frac{1}{64}$

which is required exponential form.

(iv) Given that $log_3 243 = 5$

$\Rightarrow \quad 3^5 = 243$

which is required exponential form.

(v) Given that $log_{10} 0.01 = -2$

$\Rightarrow \quad 10^{-2} = 0.01$

which is required exponential form.

Example 31. Evaluate the following:

(i) $log_2(8 \times 16)$ **(ii)** $log_4 \left(\frac{16}{256}\right)$ **(iii)** $log_3 9^3$

(iv) $log_{10} \left(\frac{1}{10}\right)^8$ **(v)** $log_e \left(\frac{1}{e^{-7}}\right)$

Ans. (i) Given expression is

$$log_2(8 \times 16) = log_2(8) + log_2(16)$$
$(used\ product\ property)$
$$= log_2 2^3 + log_2 2^4$$
$$= 3.log_2 2 + 4.log_2 2 \qquad (used\ power\ property)$$
$$= 3 + 4 = 7 \qquad (used\ log_a a = 1)$$

which is required solution.

OR

Given expression is
$$log_2(8 \times 16) = log_2(128)$$
$$= log_2 2^7 = 7 \qquad (used\ log_a a^n = n)$$

which is required solution.

(ii) Given expression is
$$log_4\left(\frac{16}{256}\right) = log_4(16) - log_4(256) \qquad (used\ quotient\ property)$$
$$= log_4 4^2 - log_4 4^4$$
$$= 2.log_4 4 - 4.log_4 4 \qquad (used\ power\ property)$$
$$= 2 - 4 = -2 \qquad (used\ log_a a = 1)$$

which is required solution.

OR

Given expression is
$$log_4\left(\frac{16}{256}\right) = log_4\left(\frac{4^2}{4^4}\right)$$
$$= log_4 4^{-2} = -2 \qquad (used\ log_a a^n = n)$$

which is required solution.

(iii) Given expression is
$$log_3 9^3 = log_3(3^2)^3$$
$$= log_3 3^6$$
$$= 6.log_3 3 \qquad (used\ power\ property)$$
$$= 6 \qquad (used\ log_a a = 1)$$

which is required solution.

(iv) Given expression is
$$log_{10}\left(\frac{1}{10}\right)^8 = log_{10}\frac{1}{10^8}$$
$$= log_{10}1 - log_{10}10^8 \qquad (used\ quotient\ property)$$
$$= 0 - 8.log_{10}10 \qquad (used\ log_a 1 = 0)$$
$$= -8 \qquad (used\ log_a a = 1)$$

which is required solution.

(v) Given expression is

$$log_e\left(\frac{1}{e^{-7}}\right) = log_e e^7$$
$$= 7. log_e e$$
$$= 7 \qquad (used\ log_a a = 1)$$

which is required solution.

Example 32. Change the base of $log_2 3$ to 10 i.e. common logarithm.

Ans. Given expression is $log_2 3$

$$= \frac{log_{10}(3)}{log_{10}(2)}$$

Example 33. Change the base of $log_7 5$ to 5.

Ans. Given expression is $log_7 5$

$$= \frac{log_5(5)}{log_5(7)} = \frac{1}{log_5(7)}$$

Example 34. Solve the equation $log_2(x+1) = log_2(x) + log_2(2x+1)$ for x.

Ans. Given equation is

$$log_2(x+1) = log_2(x) + log_2(2x+1)$$
$$\Rightarrow \quad log_2(x+1) = log_2\big(x.(2x+1)\big)$$
$$\Rightarrow \quad log_2(x+1) = log_2(2x^2+x)$$
$$\Rightarrow \quad x+1 = 2x^2 + x \qquad (used\ log_a b = log_a c\ \ iff\ \ b = c)$$
$$\Rightarrow \quad 2x^2 = 1$$
$$\Rightarrow \quad x^2 = \frac{1}{2}$$
$$\Rightarrow \quad x = \pm\sqrt{\frac{1}{2}}$$

But x can’t be negative as it is the argument of logarithm. Therefore, $= \sqrt{\frac{1}{2}}$.

Example 35. Solve the equation $log_5 a^2 = 1$ for a.

Ans. Given equation is

$$log_5 a^2 = 1$$
$$\Rightarrow \quad log_5 a^2 = log_5 5$$
$$\Rightarrow \quad a^2 = 5$$
$$\Rightarrow \quad a = \pm\sqrt{5}$$

Example 36. Solve the equation $log_{\frac{1}{2}}(y^2-1) = -1$ for y.

Ans. Given equation is

$$log_{\frac{1}{2}}(y^2-1) = -1$$
$$\Rightarrow \quad log_{\frac{1}{2}}(y^2-1) = -log_{\frac{1}{2}}\left(\frac{1}{2}\right)$$
$$\Rightarrow \quad log_{\frac{1}{2}}(y^2-1) = log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-1}$$

$\Rightarrow \quad log_{\frac{1}{2}}(y^2 - 1) = log_{\frac{1}{2}} 2$

$\Rightarrow \quad (y^2 - 1) = 2$

$\Rightarrow \quad y^2 = 3$

$\Rightarrow \quad y = \pm\sqrt{3}$

Example 37. Prove that $log_b a \,.\, log_c b \,.\, log_a c = 1$ where a, b and c are positive and are not equal to 1.

Ans. $log_b a \,.\, log_c b \,.\, log_a c$

$= \frac{loga}{logb} \cdot \frac{logb}{logc} \cdot \frac{logc}{loga} \qquad (used\ change\ of\ base\ property)$

$= 1$

Hence proved.

Example 38. Prove that $2log_2 4 + log_2 9 - log_2 6 = log_2 24$.

Ans. $2log_2 4 + log_2 9 - log_2 6$

$= log_2 4^2 + log_2 9 - log_2 6$

$= log_2 16 + log_2 9 - log_2 6$

$= log_2 (16 \times 9) - log_2 6$

$= log_2 \left(\frac{16\times 9}{6}\right)$

$= log_2 24$

Hence proved.

Example 39. Prove that $log_{10} 12 - 2log_{10} 4 + 2log_{10} 6 = log_{10} 27$.

Ans. $log_{10} 12 - 2log_{10} 4 + 2log_{10} 6$

$= log_{10} 12 - log_{10} 4^2 + log_{10} 6^2$

$= log_{10} 12 - log_{10} 16 + log_{10} 36$

$= log_{10} \left(\frac{12}{16}\right) + log_{10} 36$

$= log_{10} \left(\frac{12}{16} \times 36\right)$

$= log_{10} 27$

Hence proved.

EXERCISE-II

1. Give the examples of following:

   (i) Product property (ii) Quotient property (iii) Power property.

2. Give the examples for following:

   (i) $log_a(b.c) \neq log_a b \times log_a c$

   (ii) $log_a \left(\frac{b}{c}\right) \neq \frac{log_a(b)}{log_a(c)}$

3. Convert the following exponential forms into logarithmic forms:

   (i) $5^5 = 3125$ (ii) $(0.2)^3 = 0.008$ (iii) $3^4 = 81$

(iv) $2^{-3} = 0.125$ (v) $16^{-1} = 0.0625$ (vi) $e^0 = 1$

4. Convert the following logarithmic forms into exponential forms:

(i) $log_{100}1 = 0$ (ii) $log_{10}0.0001 = -4$ (iii) $log_e\left(\frac{1}{e}\right)^3 = -3$

(iv) $log_2 128 = 7$ (v) $log_{10}1000 = 3$

5. Evaluate the following:

(i) $log_{10}\left(\frac{1}{10^{-12}}\right)$ (ii) $log_7\left(\frac{7^5}{49}\right)$ (iii) $log_3(3^5 \times 9^3)$

(iv) $log_e\left(\frac{1}{e}\right)^{50}$ (v) $log_2\left(\frac{1}{1024}\right)$

6. Change the base of $log_5 9$ to $'e'$ i.e. natural logarithm.

7. Change the base of $log_7 19$ to $'10'$ i.e. common logarithm.

8. Change the base of $log_3 11$ to 11.

9. Solve the following equation for x:

(i) $log(2x^2 - 4) = log(2x) + log(x - 1)$

(ii) $log(x^2) = log(40) - 3log2$

(iii) $log(x) + log(6 + x) = log(16)$

(iv) $log(x^2) = log(5x - 4)$

10. Prove that the following

(i) $3log_2 4 + 4log_2 3 = log_2 5184$

(ii) $3log4 - 2log2 + 3log5 = log80 + log25$

(iii) $log_e e^6 + log_e\left(\frac{1}{e}\right)^3 = 3$

(iv) $5log_2 2 - 2log_2 3 + log_2 18 = 6$

ANSWERS

3. (i) $log_5 3125 = 5$ (ii) $log_{0.2}0.008 = 3$ (iii) $log_3 81 = 4$

(iv) $log_2 0.125 = -3$ (v) $log_{16}0.0625 = -1$ (vi) $log_e 1 = 0$

4. (i) $100^0 = 1$ (ii) $10^{-4} = 0.0001$ (iii) $e^{-3} = \left(\frac{1}{e}\right)^3$

(iv) $2^7 = 128$ (v) $10^3 = 1000$

5. (i) 12 (ii) 3 (iii) 11 (iv) -50 (v) -10

6. $\frac{log_e 9}{log_e 5}$

7. $\frac{log_{10}19}{log_{10}7}$

8. $\frac{1}{log_3 11}$

9. (i) 2 (ii) $\pm\sqrt{5}$ (iii) 2 (iv) $1, 4$

UNIT II

BINOMIAL THEOREM, DETERMINANT AND MATRICES

Learning Objectives

- Students will be able to work with permutations and combinations.
- Students will be able to use the binomial theorem to expand polynomials and to identify terms for a given polynomial.
- Students will be able to perform the matrix operations of addition, subtraction and multiplication and express a system of simultaneous linear equations in matrix form.
- Students will be able to evaluate the determinant of a 2 x 2 matrices and use the determinant to solve a system of simultaneous linear equations.

2.1 PERMUTAITON AND COMBINATION

Before knowing about concept of permutation and combination first we must be familiar with the term factorial.

Factorial : Factorial of a positive integer 'n' is defined as :

$$n! = n \times (n - 1) \times (n - 2) \ldots 3 \times 2 \times 1$$

where symbol of factorial is ! or $\lfloor\underline{\ \ }$. For example

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Note $0! = 1$

Example 1. Evaluate

(i) $6!$ (ii) $3! + 2!$ (iii) $\frac{5!3!}{2!}$ (iv) $5! + 4!$

Sol.

(i) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

(ii) $3! + 2! = 3 \times 2 \times 1 + 2 \times 1 = 6 + 2 = 8$

(iii) $\frac{5!3!}{2!} = \frac{5\times4\times3\times2\times1\times3\times2\times1}{2\times1} = 360$

(iv) 5! – 3! = 5 × 4 × 3 × 2 × 1 – 3 × 2 × 1 = 120 – 6 = 114.

Example 2. Evaluate (i) $\frac{n!}{(n-2)!}$ (ii) (n – r)! when n = 7, r = 3.

Sol. $\frac{n!}{(n-2)!} = \frac{n\times(n-1)\times(n-2)(n-3)...3\times2\times1}{(n-2)\times(n-3)...3\times2\times1}$

$= n(n-1) = n^2 - n$

(ii) (n – r)! = (7 – 3)! = 4! = 4 × 3 × 2 × 1 = 24

Example 3. Evaluate : (i) $\frac{8!-6!}{3!}$ (ii) $\frac{2!}{4!}+\frac{7!}{5!}$

Sol. (i) $\frac{8\times7\times6!-6!}{3!} = \frac{6![8\times7-1]}{3!}$

$\frac{6!\times55}{3!} = \frac{6\times5\times4\times3!\times55}{3!} = 6600$

(ii) $\frac{2!}{4!}+\frac{7!}{5!} = \frac{2!}{4\times3\times2!}+\frac{7\times6\times5!}{5!}$

$= \frac{1}{12}+\frac{42}{1} = \frac{1+504}{12} = \frac{505}{12}$

EXERCISE - VII

1. Compute $\lfloor 3 + \lfloor 6$.
2. Evaluate $\lfloor n-r$ where n = 8, r = 4.
3. Evaluate $\frac{10!}{8!3!}$.
4. Evaluate $\frac{7!-5!}{3!}$.
5. Evaluate 3!.4!.7! and prove that 3! + 4! ≠ 7!.

6. Solve the equation (n + 1)! = 12 (n – 1)!

ANSWERS

1. 726 2. 24 3. 15 4. 820

5. 3! = 6, 4! = 24, 7! = 5040 6. n = 3

Permutation: It is the number of arrangements of 'n' different things taken 'r' at a time and is calculated by the formula,

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Example 4. Evaluate $^{7}P_{4}$.

Sol. We know $^{n}P_{r} = \frac{n!}{(n-r)!}$

Put n = 7 and r = 4, we get

$$^{7}P_{4} = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1} = 840$$

Example 5. Evaluate (i) $^{6}P_{6}$ (ii) $^{4}P_{1}$

Sol. (i) $^{6}P_{6} = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6\times5\times4\times3\times2\times1}{1} = 720$.

(ii) $^{4}P_{1} = \frac{4!}{(4-1)!} = \frac{4!}{3!} = \frac{4\times3!}{3!} = 4$.

Combination: It is the grouping of 'n' different things taken 'r' at a time and is calculated by the formula.

$$^{n}c_{r} = \frac{n!}{(n-r)!r!}$$

Example 6. Evaluate : (i) $^{9}c_{5}$ (ii) $^{n}c_{0}$ (iii) $^{n}c_{n}$

Sol. (i) As $^{n}c_{r} = \frac{n!}{(n-r)!r!}$

Put n = 9, r = 5, we get

$$^{9}c_5 = \frac{9!}{(9-5)!5!} = \frac{9!}{4!5!} = \frac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!} = 126$$

(ii) $$^{n}c_0 = \frac{n!}{(n-0)!0!} = \frac{n!}{n!\,0!} = 1 \quad \text{as } 0! = 1$$

(iii) $$^{n}c_n = \frac{n!}{(n-n)!n!} = \frac{n!}{0!n!} = 1$$

EXERCISE - I

1. Define permutation and combination with examples.

2. if n = 10, r = 4 then find value $\frac{n!}{(n-r)!}$.

3. Evaluate (i) $^{10}P_2$ (ii) $^{5}P_5$ (iii) $^{8}c_3$ (iv) $^{6}c_0$

4. Find n if $^{n}p_2 = 20$.

5. Find the value of $^{10}c_3 + ^{10}c_4$.

6. If 11. $^{n}P_4 = 20.\ ^{n}P_2$ then the value of n is

   (a) 7 (b) –7 (c) 16 (d) None of these

7 - The value of 1.3.5……(2n -1) . 2^n equals

   (a) $\frac{(2n)!}{2^n}$ (b) $\frac{n!}{2^n}$ (c) $\frac{(2n)!}{n!}$ (d) None of these

8 - If $^{n}C_{r-1} = 36$, $^{n}C_r = 84$, $^{n}C_{r+1} = 126$ then r is equal to

   (a) 3 (b) 2 (c) 1 (d) None of these

9 - The value of 1 . 1! + 2 . 2! + 3 . 3! + 4 . 4! is

   (a) 118 (b) 119 (c) 120 (d) None of these

ANSWERS

2. 5040 3. (i) 90 (ii) 1 (iii) 56 (iv) 1 4. 5 5. 330

6. (c) 7. (c) 8. (a) 9. (b)

BINOMIAL THEOREM

Binomial Theorem for Positive Integer: If n is any positive integer, then

$(x+a)^n = {}^n c_0 x^{n-0} a^0 + {}^n c_1 x^{n-1} a^1 + {}^n c_2 x^{n-2} a^2 + {}^n c_n x^{n-n} a^n$ is called Binomial expansion, where ${}^n c_0, {}^n c_1, {}^n c_2, ..., {}^n c_n$ are called Binomial co-efficients.

Features of Binomial Theorem :

(i) The number of terms in Binomial expansion is one more than power of Binomial expression.

(ii) In Binomial expansion the sum of indices of x and a is equal to n.

(iii) The value of Binomial co-efficient, equidistant from both ends is always same.

Application in Real Life :

In real life Binomial theorem is widely used in modern world areas such a computing, i.e. Binomial Theorem has been very useful such as in distribution of IP addresses. Similarly in nation's economic prediction, architecture industry in design of infrastructure etc.

Example 7. How may terms are there in binomial expansion of $(a + b)^7$.

Sol. The number of terms in binomial expansion of $(a + b)^7$ is (n + 1) where n = 7. So total number of terms in expansion = 8.

Example 8. Which of the binomial co–efficients have same value in $(x + a)^7$ ${}^7c_0, {}^7c_1, {}^7c_2, {}^7c_3, {}^7c_4, {}^7c_5, {}^7c_6, {}^7c_7$.

Sol. ${}^7c_0 = {}^7c_7 = 1,$ ${}^7c_1 = {}^7c_6 = 7$

${}^7c_2 = {}^7c_5 = 21$ ${}^7c_3 = {}^7c_4 = 35$

Example 9. Expand $(x + y)^7$ binomially.

Sol. $(x+y)^7 = {}^7 c_0 x^7 y^0 + {}^7 c_1 x^6 y^1 + {}^7 c_2 x^5 y^2 + {}^7 c_3 x^4 y^3 + {}^7 c_4 x^3 y^4$

$$+{}^7c_5x^2y^5+{}^7c_6x^1y^6+{}^7c_7x^0y^7$$

(1)

as $\quad {}^7c_0={}^7c_7=1, \quad {}^7c_1={}^7c_6=7$

$\quad {}^7c_2={}^7c_5=21 \quad {}^7c_3={}^7c_4=35$

so equation (1) becomes

$$(x+y)^7=x^7+7x^6y^1+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6+y^7$$

EXERCISE - II

1. State Binomial Theorem for n as a positive integer.
2. Write the number of terms in expansion of $(x + y)^{10}$.
3. Which of Binomial co–efficients in binomial expansion of $(a + b)^8$ have same value. Also evaluate.
4. Expand $(x + 2y)^5$ using Binomial theorem.
5. Expand $\left(x+\frac{1}{x}\right)^6$ using Binomial Theorem.
6. Expand $(2x – 3y)^4$ using Binomial Theorem.
7. Expand $(a^2 + b^3)^4$ using Binomial Theorem.

ANSWERS

2. 11 $\qquad$ 3. ${}^8c_0={}^8c_8=1, {}^8c_1={}^8c_7=8, {}^8c_2={}^8c_6=28, {}^8c_3={}^8c_5=56, {}^8c_4=70$

4. $(x+2y)^5=x^5+10x^4y+40x^3y^2+80x^2y^3+80xy^4+32y^5$

5. $\left(x+\frac{1}{x}\right)^6=x^6+6x^4+15x^2+20+\frac{15}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$

6. $(2x-3y)^4=16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$

7. $(a^2+b^3)^4 = a^8+4a^6b^3+6a^4b^6+4a^2b^9+b^{12}$

General Term of Binomial Expression $(x + a)^n$ for positive integer 'n'

$$T_{r+1} = {}^nc_r x^{n-r} a^r \qquad \text{where } 0 \le r \le n$$

Generally we use above formula when we have to find a particular term.

Example 10. Find the 5^{th} term in expansion of $(x - 2y)^7$.

Sol. Compare $(x - 2y)^7$ with $(x + a)^n$

$\Rightarrow$ $x = x$, $a = -2y$, $n = 7$

$T_5 = T_{r+1} \Rightarrow r + 1 = 5, \quad r = 4$

Using $T_{r+1} = {}^nc_r x^{n-r} a^r$

Put all values

$$T_{4+1} = {}^7c_4 x^{7-4}(-2y)^4$$

$$= \frac{7!}{3!4!} x^3 16y^4$$

$$= 560\, x^3\, y^4$$

Note : p^{th} term from end in expansion of $(x + a)^n$ is $(n - p + 2)^{th}$ term from starting.

To find Middle term in $(x + a)^n$ when n is positive integer

(a) when n is even positive integer,

Middle term = $T_{\frac{n}{2}+1}$

(b) When n is odd positive integer

Middle term = $T_{\frac{n+1}{2}}$ and $T_{\frac{n+1}{2}+1}$

Example 11. In Binomial expression (i) $(x + y)^{10}$ (ii) $(x + y)^{11}$. How many middle terms are there.

Sol. (i) In $(x + y)^{10}$

This binomial expression has only one Middle Term

i.e. $T_{\frac{10}{2}+1} = T_6$

(ii) In $(x + y)^{11}$

This Binomial expression has two middle terms

i.e. $T_{\frac{11+1}{2}}$ and $T_{\frac{11+1}{2}+1}$

or T_6 and T_7.

Example 12. The 3^{rd} term from end in binomial expansion of $(x - 2y)^7$ is _____ term from starting.

Sol. Using formula $(n - p + 2)^{th}$ term.

Here $n = 7$, $p = 3$.

So 3^{rd} term end in binomial expansion of $(x - 2y)^7$ is $(7 - 3 + 2)^{th}$ term from starting, i.e. 6^{th} from starting.

Example 13. Find the middle term in expansion of $\left(\frac{x}{y} + \frac{y}{x}\right)^{12}$.

Sol. Middle term = $T_{\frac{12}{2}+1} = T_7$

Compare $\left(\frac{x}{y} + \frac{y}{x}\right)^{12}$ with $(x + a)^n$

$$x = \frac{x}{y}, \qquad a = \frac{y}{x}, \qquad n = 12$$

Using $T_{r+1} = {}^nC_r x^{n-r} a^r$

$T_{r+1} = T_7 \quad \Rightarrow \quad r = 6$

$$T_{6+1} = {}^{12}C_6 \left(\frac{x}{y}\right)^{12-6} \left(\frac{y}{x}\right)^6$$

$$= 924 \left(\frac{x}{y}\right)^6 \left(\frac{y}{x}\right)^6$$

$T_{6+1} = 924$

Example 14. Find 5^{th} term from end in expansion of $\left(\frac{x^2}{2} - \frac{2}{x^3}\right)^9$.

Sol. Compare $\left(\frac{x^2}{2} - \frac{2}{x^3}\right)^9$ with $(x + a)^n$

$x = \frac{x^2}{2}$, $\quad a = -\frac{2}{x^3}$, $\quad n = 9$

5^{th} term from end is $(n - p + 2)^{th}$ term from staring.

i.e. $(9 - 5 + 2)^{th}$ term from starting.

6^{th} term from staring.

Using $T_{R+1} = {}^nC_r x^{n-r} a^r$

$T_{R+1} = T_6 \quad \Rightarrow \quad r + 1 = 6, \ r = 5.$

So, $$T_{5+1} = {}^9C_5 \left(\frac{x^2}{2}\right)^{9-5} \left(-\frac{2}{x^3}\right)^5$$

$$= -126 \frac{x^8}{16} \times \frac{32}{x^{15}}$$

$$T_{5+1} = \frac{-252}{x^7}$$

EXERCISE - III

1. Find 4th term of $\left(x+\frac{1}{x}\right)^7$ Binomially.

2. Find the 4th term of $\left(\frac{4x}{7}-y^2\right)^5$ Binomially.

3. Find middle term of $(x-2y)^7$ using Binomial theorem.

4. Find 5th term in Binomial expansion $\left(x^2+\frac{1}{x}\right)^7$.

5. Find 3rd term from end in Binomial expansion $(2x-3)^6$.

6. Find Middle Term in Binomial expansion $\left(\frac{2}{x}-\frac{x}{2}\right)^5$.

7. In Binomial expansion of $\left(\frac{4x}{7}-y^2\right)^5$. Find 4th term.

8. The total number of terms in the expansion of $(x+y)^{100}$ is
   (a) 100 (b) 200 (c) 101 (d) None of these

9. The coefficient of x^5 in expansion of $\left(x+\frac{1}{x^3}\right)$ is
   (a) 685 (b) 680 (c) 780 (d) None of these

10. The constant term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is
   (a) 152 (b) -152 (c) 252 (d) -252

11 - In the expansion of $(3x+2)^4$, the coefficient of middle term is
   (a) 95 (b) 64 (c) 236 (d) 216

ANSWERS

1. $35x$ 2. $-\frac{160}{49}x^2y^6$ 3. (i) $-280x^4y^3$ (ii) $560x^3y^4$ 4. $35x^2$

5. $4860x^2$ 6. (i) $\frac{20}{x}$ (ii) $-5x$ 7. $-\frac{160}{49}x^2y^6$ 8. (c) 9. (a)

10. (d) 11. (d)

Binomial theorem for any Index

Let n be a any rational number, positive or negative, integral or fractional and x be any real number such that | x | < | then

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots\ldots\ldots\ldots + \frac{n(n-1)\ldots\ldots\ldots(n-r+1)x^r}{r!} + \ldots\ldots\ldots\ldots \propto$

Note (1) Number of terms in $(1+x)^n$ is infinite

(2) The first term in $(1+x)^n$ should be unity

Some particular expansion –

(1) $(1+x)^{n} = 1 + (-n)x + \frac{(-n)(-n-1)x^2}{2!} + + \frac{(-n)(-n-1)(-n-2)x^3}{3!} + \ldots$

$= 1 - nx + \frac{n(n+1)x^2}{2!} - \frac{n(n+1)(n+2)}{3!}x^3 + \ldots \propto$

(2) $(1-x)^n = 1 + n(-x) + \frac{n(n-1)}{2!}(-x)^2 + \ldots\ldots\ldots\ldots \propto$

$= 1 - nx + \frac{n(n-1)x^2}{2!} + \ldots\ldots\ldots\ldots\ldots\ldots \propto$

Expample 15. Expand $(1+x)^{-3}$ upto first three terms

Sol:- $(1+x)^{-3} = 1 + (-3)x + \frac{(-3)(-3-1)x^2}{2!} + \ldots\ldots$

$= 1 - 3x + \frac{12}{2!}x^2 + \ldots\ldots\ldots$

$= 1 - 3x + 6x^2 + \ldots\ldots\ldots$

Expample 16. Expand $(1-x)^{-\frac{3}{2}}$ upto first three terms

Sol:- $(1-x)^{-\frac{3}{2}} = 1 + (-\frac{3}{2})(-x) + \frac{\left(-\frac{3}{2}\right)\left(-\frac{3}{2}-1\right)(-x)^2}{2!} + \ldots\ldots$

$= 1 + \frac{3x}{2} + \frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)x^2}{2!} + \ldots\ldots$

$= 1 + \frac{3x}{2} + \frac{15}{8}x^2 + \ldots\ldots$

Expample 17. Expand $(2-3x)^{-3}$ upto first three terms

Sol:- $(2-3x)^{-3} = (2)^{-3}(1-\frac{3x}{2})^{-3} = \frac{1}{8}(1-\frac{3x}{2})^{-3}$

$\frac{1}{8}(1-\frac{3x}{2})^{-3} = \frac{1}{8}(1 + (-3)(-\frac{3x}{2}) +$

$\frac{(-3)(-4)}{2!}(\frac{-3x}{2})^2 + \ldots\ldots\ldots\ldots)$

$= \frac{1}{8}(1 + \frac{9x}{2} + (\frac{12}{2})(\frac{9x^2}{4}) + \ldots\ldots\ldots\ldots)$

$= \frac{1}{8}(1 + \frac{9x}{2} + \frac{27}{2}x^2 + \ldots\ldots\ldots\ldots)$

Expample 18. Expand $(4+3x)^{-2}$ upto first three terms

Sol:- $(4+3x)^{-2} = (4)^{-2}(1+\frac{3x}{4})^{-2} = \frac{1}{2}(1+\frac{3x}{4})^{-2}$

$= \frac{1}{2}(1 + (-2)(\frac{3x}{4}) + \frac{(-2)(-3)}{2!}\left(\frac{3x}{4}\right)^2 + \ldots\ldots\ldots)$

$= \frac{1}{2}\left(1- \frac{6x}{4}+\left(\frac{6}{2}\right)\left(\frac{9x^2}{16}\right) + \dots\dots\dots\dots\right)$

$= \frac{1}{2}\left(1- \frac{3x}{2}+\frac{27}{16}x^2 + \dots\dots\dots\dots\right)$

$= \frac{1}{2} - \frac{3x}{4}+\frac{27}{32}x^2 + \dots\dots\dots\dots$

Expample 19. Find the cube roof of 998 correct to five place of decimal

Sol:- $(998)^{\frac{1}{3}} = (1000-2)^{\frac{1}{3}} = (1000)^{\frac{1}{3}}\left(1-\frac{2}{1000}\right)^{\frac{1}{3}}$

$= 10\left(1-\frac{2}{1000}\right)^{\frac{1}{3}}$

$=10\left[1+\frac{1}{3}\left(-\frac{2}{1000}\right) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}\left(-\frac{2}{1000}\right)^2 \dots\dots\dots\dots\right]$

$=10\,[1 - .0006666 - .0000004]$

= 9.99334(approx)

<u>Binomial Approximation</u>

(Application of Binomial theorem)

(1) Ist Binomial Approximation – If x is numerically so small that its square and higher power may be neglected, then

$(1+x)^n = 1+nx$ (Approximately)

[$\therefore x^2, x^3, \dots\dots$ are all approximately zero

(2) Second Binomial Approximation – If x be numerically so small that its cube and higher power may be neglected, then

$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2$(Approximately)

[$Here\ \ x^3, x^4, \dots\dots$ are all approximately zero

Expample 20. If x be numerically so small that its cube and higher power may be neglected, then find the binomial approximation of

$(1+3x)^{-2}$

Sol:- $(1+3x)^{-2} = 1+(-2)(3x)+\frac{(-2)(-3)}{2!}(3x)^2$

(Neglected $x^3,\ x^4, \dots\dots$

$= 1- 6x + \frac{6}{2}(9x^2)$

$= 1-6x+27x^2$

Expample 21. If x is numerically so small that x^2 and higher power may be neglected, then prove that

$\frac{(1-2X)^{\frac{3}{2}}(4+5X)^{\frac{3}{2}}}{\sqrt{1-X}} = 8 +\frac{25}{3}x$

Sol:- We have to expand each binomial expansion up to first two terms only

LHS $\frac{(1-2X)^{\frac{2}{3}}(4+5X)^{\frac{3}{2}}}{(1-x)^{\frac{1}{2}}} = \frac{(1-2X)^{\frac{2}{3}}(4)^{\frac{3}{2}}\left(1+\frac{5x}{4}\right)^{\frac{3}{2}}}{(1-x)^{\frac{1}{2}}}$

$= 8\frac{(1-2X)^{\frac{2}{3}}(1+\frac{5x}{4})^{\frac{3}{2}}}{(1-x)^{\frac{1}{2}}}$

$= 8(1-2x)^{\frac{2}{3}}(1+5x)^{\frac{3}{2}}(1-x)^{\frac{-1}{2}}$

$= 8[1+\frac{2}{3}(-2x)]\ [1+\frac{3}{2}(\frac{5x}{4})]\ [1+(-\frac{1}{2})(-x)]$

$= 8(1-\frac{4x}{3})\left(1+\frac{15x}{8}\right)(1+\frac{x}{2})$

$= 8\ (1+\frac{15x}{8}-\frac{4x}{3})\left(1+\frac{x}{2}\right)$

$= 8\ (1+\frac{13x}{24})\ (1+\frac{x}{2})$

$= 8\ (1+\frac{x}{2}+\frac{13x}{24})\ [neglecting\ term\ of\ x^2]$

$= 8\ (1+\frac{25}{24}x)$

$= 8+\frac{25}{3}x = RHS$

Exercise - IV

(1) Expand the following upto first three terms

i.) $(1-x)^{-2}$ (Ans:- $1+2x+3x^2+\ldots\ldots\ldots$)

ii.) $(1-2x^3)^{\frac{11}{2}}$ (Ans:- $1-11x^3+\frac{99}{2}x^6-\ldots\ldots.$)

iii.) $(1+2x)^{\frac{1}{3}}$ (Ans:- $1+\frac{2x}{3}-\frac{4x^2}{9}+\ldots\ldots\ldots$)

iv.) $(2-3x)^{\frac{5}{2}}$ (Ans:- $2^{\frac{5}{2}}(1-\frac{15x}{4}+\frac{135x^2}{32}-\frac{135x^3}{128}+\ldots\ldots\ldots.$)

v.) $(27-6x)^{\frac{-2}{3}}$ (Ans:- $\frac{1}{9}(1+\frac{4x}{27}+\frac{20x^2}{729}+\ldots\ldots\ldots\ldots$)

Q:-2 Use binomial theorem to evaluate

1. $(244)^{\frac{1}{5}}$ correct to three places of decimal. (Ans:- 3.0041)
2. $(1.025)^{-\frac{1}{3}}$ correct to three places of decimal (Ans:- 0.9918)

.

Q:-3If x is so small that square and higher power may be neglected, then prove that

1. $(1+\frac{3x}{4})^{-4}\ (4-3x)^{\frac{1}{2}} = 2-\frac{27}{4}x$
2. $\frac{(9+7x)^{\frac{1}{2}}-(16+3)^{\frac{1}{4}}}{(4+5x)} = \frac{1}{4}-\frac{17}{384}x$
3. $\frac{(1-2x)^{\frac{1}{2}}+(1+3x)^{\frac{4}{3}}}{(3+x)+(4-x)^{\frac{1}{2}}} = \frac{2}{5}+\frac{27}{50}x$

2.2 DETERMINANTS AND MATRICES

Determinant : The arrangement of n^2 elements between two vertical lines in n rows and n–columns is called a determinant of order n and written as

$$D = \begin{vmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & \cdots & a_{nn} \end{vmatrix}_{n\times n}$$

Here a_{11}, a_{12}, a_{13}, … a_{nn} are called elements of determinant.

The horizontal lines are called rows and vertical lines are called columns.

Here $a_{11} \quad a_{12} \quad \ldots \quad a_{1n} \quad \rightarrow$ R_1 (First Row)

$a_{21} \quad a_{22} \quad \ldots \quad a_{2n} \quad \rightarrow$ R_2(IInd Row)

and

$$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \cdots & \cdots \\ a_{n1} & a_{n2} \\ \downarrow & \downarrow \\ \text{Ist column} & \text{2nd column} \\ (C_1) & (C_2) \end{matrix}$$

Determinant of order 2: The arrangement of 4 elements in two rows and two columns between two vertical bar is called a determinant of order 2.

i.e.

$$\begin{matrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{matrix} \rightarrow R_1 \text{ (first row)} \\ \rightarrow R_2 \text{ (2nd row)} \end{matrix} \\ \begin{matrix} C_1 & C_2 \\ \text{(1st) Col.} & \text{(2nd) Col.} \end{matrix} & \end{matrix}$$

Here a_{11}, a_{12}, a_{21}, a_{22} are called elements of determinant

Value of Determinant of order 2 :

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$ Multiplying diagonally with −sign in downward arrow

$= a_{11}\, a_{22} - a_{21}\, a_{12}$

Example 22. Solve $D = \begin{vmatrix} 2 & 5 \\ 3 & 9 \end{vmatrix}$.

Sol. $D = \begin{vmatrix} 2 & 5 \\ 3 & 9 \end{vmatrix} = 2(9) - 3(5) = 18 - 15 = 3$

Example 23. Find the value of, $D = \begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$.

Sol. $D = \begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta - (-\cos^2\theta) = \sin^2\theta + \cos^2\theta = 1.$

Minor of an element

Definition: A minor of an element in a determinant in obtained by deleting row and column in which that element occurs.

Note: Minor of an element a_{ij} is denoted by M_{ij}

Let Determinant of order 2:

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\text{Minor of } a_{11} = M_{11} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{22}$$

Minor of $a_{12} = M_{12} = a_{21}$

Minor of $a_{21} = M_{21} = a_{12}$

Minor of $a_{22} = M_{22} = a_{11}$

Example 24. Find minor of each element in determinant $D = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

Minor of 1 = 4

Minor of 2 = 3

Minor of 3 = 2

Minor of 4 = 1

Example 25. Find x if $\begin{vmatrix} 4 & x \\ x & 4 \end{vmatrix} = 0$.

Sol. Given that $\begin{vmatrix} 4 & x \\ x & 4 \end{vmatrix} = 0$

$\Rightarrow$ $16 - x^2 = 0$ $\Rightarrow$ $x^2 = 16$, $x = \pm 4$

Example 26. If $\begin{vmatrix} 3 & 2 \\ x & 6 \end{vmatrix} = 0$, find value of x.

Sol. Given that $\begin{vmatrix} 3 & 2 \\ x & 6 \end{vmatrix} = 0$

$\Rightarrow$ $18 - 2x = 0$ $\Rightarrow$ $-2x = -18$, $\Rightarrow$ $x = 9$

Solution of Equations by Cramer's Rule (having 2 unknown)

By Cramer's Rule, we can solve simultaneous equations with unknown using Determinants.

Solve the following equation by Cramer's rule

$$a_1x + b_1y = C_1 \quad (1)$$

$$a_2x + b_2y = C_2 \quad (2)$$

Solution of eqn. (1) and (2) by Cramer's Rule is

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}$$

where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ (Replacing first column by constants)

$D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ (Replacing by 2^{nd} column by constants)

Note : (1) The equations have unique solution, if $D \neq 0$.

(2) The equations have an infinite number of solutions if $D = D_1 = D_2 = 0$.

(3) The equations have no solution if $D = 0$ and any one of D_1 or D_2 is not zero.

Consistent: When a system of equations has a solution, then equations are said to be consistent.

Inconsistent: If equations have no solution, then equations are said to be inconsistent.

Example 27. Solve the system of equations using Cramer's rules:

$$x + 2y = 1$$

$$3x + y = 4$$

Sol. $D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = -5$

$$D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = 1 - 8 = -7$$

$$D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 4 - 3 = 1$$

$\therefore$ $x = \frac{D_1}{D} = \frac{-7}{-5} = \frac{7}{5}$

$$y = \frac{D_2}{D} = \frac{1}{-5} = -\frac{1}{5}$$

EXERCISE -V

1. Find minors of all elements in $\begin{vmatrix} 7 & -3 \\ 4 & 2 \end{vmatrix}$.

2. Find minors and co–factors of all elements of determinant $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$.

3. Evaluate $\begin{vmatrix} 2 & 4 \\ -5 & 1 \end{vmatrix}$.

4. Evaluate $\begin{vmatrix} 1 & \sin\theta \\ \sin\theta & 1 \end{vmatrix}$

5. Find x if $\begin{vmatrix} 4 & x \\ x & 4 \end{vmatrix} = 0$.

6. Solve by Cramer's rule $x + 3y = 4$

   $4x - y = 3$

ANSWERS

1. $M_{11} = 2$, $M_{12} = 4$, $M_{21} = -3$, $M_{22} = 7$

2. $M_{11} = 3$, $M_{12} = 0$, $M_{21} = -4$, $M_{22} = 2$; $C_{11} = 3$, $C_{12} = 0$, $C_{21} = 4$, $C_{22} = 2$

3. 22 4. $\cos^2\boldsymbol{\theta}$ 5. $x = \pm 4$ 6. $x = 1$, $y = 1$

MATRICES

Matrix: The arrangement of m × n elements in m–row and n–columns enclosed by a pair of brackets [ ] is called a matrix of order m × n. The matrix is denoted by capital letter A, B, C etc.

A matrix of order m × n is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m\times n}$$

i.e. Matrix have m rows and n columns. In short we can write it as

$$A = [a_{ij}]$$

where $i = 1, 2, 3, \ldots, m \leftarrow$ row

$j = 1, 2, 3, \ldots, n \leftarrow$ columns

Note – Difference between a determinant & a matrix is that a determinant is always in square form [i.e. m = n], but matrix may be in square or in rectangular form. Determinant has a definite value, but matrix is only arrangement of elements with no value.

Order of Matrix: Number of rows × number of column

e.g. $$A = \begin{bmatrix} 2 & 1 \\ 2 & 5 \end{bmatrix}_{2\times 2}$$ is a matrix of order 2 × 2

$$B = \begin{bmatrix} 2 & 5 \\ 1 & 6 \\ -2 & 0 \end{bmatrix}_{3\times 2}$$ is a matrix of order 3 × 2

Types of Matrices:

(1) Square Matrix: A matrix is said to be a square matrix if number of rows of matrix is equal to number of columns of Matrix i.e. m = n.

For ex (i) $$\begin{bmatrix} 2 & 0 \\ 5 & 7 \end{bmatrix}_{2\times 2}$$ is a square matrix of order 2 × 2.

(ii) $$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 1 \\ 0 & 5 & 6 \end{bmatrix}_{3\times 3}$$ is a square matrix of order 3 × 3

(2) Rectangular Matrix: A matrix where, number of rows is not equal to number of columns i.e. $m \neq n$

e.g. $$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 0 & 6 \end{bmatrix}_{2\times 3}$$ is a rectangular matrix of order 2 × 3

(3) Row Matrix: A matrix having one row and any number of columns is called row matrix

e.g. (i) $A = [-3 \quad 2]_{1 \times 2}$ is a row matrix of order 1×2

(ii) $B = [5 \quad 7 \quad -2]_{1 \times 3}$ is a row matrix of order 1×3

(4) Column Matrix: A matrix having only one column and any number of rows is called column matrix.

e.g. $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}_{2\times1}$ order of matrix is 2×1

$B = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}_{3\times1}$ order of matrix is 3×1

(5) Diagonal Matrix: A matrix is to be a diagonal matrix if all non–diagonal elements are zero.

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$

(6) Null Matrix: A matrix whose all elements are zero is called a null matrix. It is denoted by 0.

e.g. $0_{2x2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $0_{3x3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(7) Unit matrix: A diagonal matrix each of whose diagonal element is equal to unity is called unit matrix

For example, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are unit matrices of order 2 and 3 respectively.

(8) Scalar Matrix: A diagonal matrix is said to be scalar matrix of all diagonal elements are equal.

e.g. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(9) Upper Triangular Matrix: A square matrix is which all the elements below the principle diagonal are zero is called an upper triangular matrix.

e.g. $\begin{bmatrix} 2 & 3 & 7 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

(10) Lower Triangular Matrix: A square matrix in which all the elements above the principal diagonal are zero is called lower triangular matrix.

e.g. $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 5 & 0 \\ 1 & 3 & 2 \end{bmatrix}$

(11) Equal Matrix: Two matrices are said to equal if they have same order and their corresponding elements are identical

For example $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

If $x_1 = 2$, $x_2 = 4$, $x_3 = 6$, $x_4 = 8$

(12) Transpose of Matrix: A matrix obtained by interchanging its rows and columns is called Transpose of the given matrix. Transpose of A is denoted by A^T or A'.

e.g. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 5 & 6 \end{bmatrix}$

then $A^T = \begin{bmatrix} 2 & 2 \\ 3 & 5 \\ 1 & 6 \end{bmatrix}$

Note $(A^T)^T = A$.

(13) Symmetric Matrix: A matrix is said to be symmetric if it is equal to its transpose i.e. $A^T = A$.

For ex $$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$ is a symmetric matrix

(14) Skew–symmetric matrix: A square matrix is said to be skew–symmetric is

$$A^T = -A$$

Note : The diagonal elements of skew–symmetric matrix are always be zero.

For ex $$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$ is skew symmetric matrix

(15) Singular Matrix: A square matrix is said to be singular if $|A| = 0$. *i.e.* determinant, where $|A|$ is the determinant of matrix A.

(16) Non–singular matrix: A matrix is said to be non–singular if $|A| \neq 0$.

Formation of a Matrix/Construction of Matrix:

Example 28. Construct a 2×2 matrix whose elements are $a_{ij} = i + j$.

Sol. We have

$$a_{ij} = i + j$$

Required matrix of 2×2 is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

Example 29. Construct a matrix of 2×2 whose element is given by $a_{ij} = \frac{(i+2j)^2}{2}$.

Sol. We have

$$\therefore \quad a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$a_{22} = \frac{(2+2(2))^2}{2} = \frac{36}{2} = 18$$

$\therefore$ $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Example 30. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ find the values of a and b.

Sol. Given matrices are equal, therefore their corresponding elements are identical

$\therefore$ $a + b = 6 \quad \Rightarrow \quad a = 6 - b$

and $ab = 8$

$\Rightarrow$ $(6 - b)b = 8$

$6b - b^2 = 8 \quad \Rightarrow \quad -b^2 + 6b - 8 = 0$

$\Rightarrow$ $b^2 - 6b + 8 = 0 \Rightarrow \quad (b - 2)(b - 4) = 0$

$\Rightarrow$ $b = 2$ and $b = 4$

If $b = 2 \quad \Rightarrow \quad a = 6 - b = 6 - 2 = 4$

If $b = 4 \quad \Rightarrow \quad a = 6 - b = 6 - 4 = 2$

$\therefore$ $a = 2,\ b = 4$ & $a = 4,\ b = 2$

Example. 31. Find the value of x, y, z and a if $\begin{bmatrix} 2x+3 & x+2y \\ z+3 & 2a-4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$.

Sol. given matrices are equal

$\therefore$ $2x + 3 = 1$ (i)

$x + 2y = 2$ (ii)

$z + 3 = -1$ (iii)

$2a - 4 = 3$ (iv)

From eqn. (i) $2x = 1 - 3 = -2$

$2x = -2 \quad \Rightarrow \quad x = -1$

Substitute x = −1 in (ii)

$-1 + 2y = 2$

$2y = 3 \quad \Rightarrow \quad y = \frac{3}{2}$

From eqn. (iii)

$z + 3 = -1 \quad \Rightarrow \quad z = -4$

From eqn. (iv)

$2a - 4 = 3$

$2a = 7 \quad \Rightarrow \quad a = \frac{7}{2}$

$\therefore \quad x = -1,\ y = \frac{3}{2},\ z = -4,\ a = \frac{7}{2}$

Algebra of Matrices: (Addition, Subtraction, and Multiplication of Matrices)

Addition of Matrices: If A and B are two matrices having same order, then their addition A + B is obtained by adding there corresponding elements

For example, If

$$A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \\ 7 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 5 & 7 \\ 0 & 6 \\ 2 & 3 \end{bmatrix}$$

Then
$$A + B = \begin{bmatrix} 2+5 & 5+7 \\ 6+0 & 8+6 \\ 7+2 & 0+3 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 6 & 14 \\ 9 & 3 \end{bmatrix}$$

Properties of Matrix addition:

If A, B and C are three matrix of same order then

(i) A + B = B + A (commutative law)

(ii) A + (B + C) = (A + B) + C [Associative law)

(iii) A + 0 = 0 + A, where **0** is null matrix

(iv) A + (–A) = (–A) + A = **0**

Here (–A) is called additive inverse of matrix A.

Subtraction of Matrices: If A and B are two matrices of same order, then A − B is obtained by subtracting element of B from the corresponding elements of A.

For example, Let $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$

then $A - B = \begin{bmatrix} 5-3 & 2-5 \\ 3-2 & 2-0 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$

Note : A – B ≠ B – A.

Scalar Multiplication: The matrix obtained by multiplying each element of a given matrix by a scalar K.

If $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$, then scalar multiplication of A by the scalar 2 is given by

$$2A = 2\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 8 \end{bmatrix}$$

Multiplication of Two Matrices:

If A and B are two matrices, then their product AB is possible only **if number of columns in A is equal to number of rows in B.**

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$

Then $C = [c_{ij}]_{m \times p}$ is called the product of A and B.

Example 32. If $A = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$, find AB.

Sol.

$$AB = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix} = \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \end{bmatrix}$$

$$= \begin{bmatrix} 8-8 & 24-24 \\ 16-16 & 48-48 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Note : $AB \neq BA$ (In general)

Properties of Multiplication of Matrices

(i) $AB \neq BA$ (in general)

(ii) $A(B + C) = AB + AC$

(iii) $AI = A$, where I is the unit matrix.

Power of a Matrix: If A is a square matrix *i.e.* number of rows = number of its columns then,

$$A^2 = A.A$$

$$A^3 = A^2.A = A.A^2$$

Similarly we can find other power of square matrix.

e.g. if $A = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$, then find A^2

$$A^2 = A.A = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2+0 \\ -2+0 & -4+0 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & -4 \end{bmatrix}$$

Example 33. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, then find 5A and 3B.

Sol. We have $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, then

$$5A = 5\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 20 & 35 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 12 & 18 \end{bmatrix}$$

Example 34. If $A = \begin{bmatrix} 7 & 3 \\ -5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 \\ 5 & 8 \end{bmatrix}$, then find A – B.

Sol. $$A - B = \begin{bmatrix} 7 & 3 \\ -5 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 9 & -1 \\ -10 & -1 \end{bmatrix}$$

Example 35. If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, then find 2A – 3B.

Sol. We have

$$A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}, \ 2A = 2\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ -2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, \ 3B = 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 9 & 6 \end{bmatrix}$$

$$2A - 2B = \begin{bmatrix} 10-6 & 6-(-3) \\ -2-9 & 2-6 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ -11 & -4 \end{bmatrix}$$

Example 36. If $X = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $Y = \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$. Find 3X + Y.

Sol. $$3X = 3\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}$$

$$3X + Y = \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 6+5 \\ -9+1 & 12-3 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$$

Example 37. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find 2A + 3B + 5I, where I is a unit matrix of order 2.

Sol. $$2A = 2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

$$5I = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Now $$2A + 3B + 5I = \begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3+5 & 6+9+0 \\ 8-6+0 & 14+15+5 \end{bmatrix} = \begin{bmatrix} 12 & 15 \\ 2 & 34 \end{bmatrix}$$

Example 38. if $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$, show that AB = BA.

Sol. $$AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2(3)+5(-1) & 2(-5)+5(2) \\ 1(3)+3(-1) & 1(-5)+3(2) \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & 15-15 \\ -2+2 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ AB = BA

Example 39. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -2 & 4 \end{bmatrix}$, show that $(A + B)^2 = A^2 + 2AB + B^2$.

Sol. $A + B = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ -4 & 4 \end{bmatrix}$

LHS $(A + B)^2 = (A + B)(A + B) = \begin{bmatrix} -2 & 4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ -4 & 4 \end{bmatrix}$

$$= \begin{bmatrix} -12 & 8 \\ -8 & 0 \end{bmatrix} \qquad (1)$$

$$A^2 = A.A. = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 1-4 & 2+0 \\ -2+0 & -4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ -2 & -4 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -3-4 & 2+8 \\ 6+0 & -4+0 \end{bmatrix} = \begin{bmatrix} -7 & 10 \\ 6 & -4 \end{bmatrix}$$

$$2AB = 2\begin{bmatrix} -7 & 10 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} -14 & 20 \\ 12 & -8 \end{bmatrix}$$

$$B^2 = B.B. = \begin{bmatrix} -3 & 2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} +9-4 & -6+8 \\ 6-8 & -4+16 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ -2 & 12 \end{bmatrix}$$

$\therefore$ RHS $A^2 + 2AB + B^2$

$$= \begin{bmatrix} -3 & 2 \\ -2 & -4 \end{bmatrix} + \begin{bmatrix} -14 & 20 \\ 12 & -8 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} -3-14+5 & 2+20+2 \\ -2+12-2 & -4-8+12 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 24 \\ 8 & 0 \end{bmatrix}$$

$\therefore$ LHS $\neq$ RHS

Example 40. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 \\ 4 & 5 \end{bmatrix}$. Verify $(AB)^T = B^T A^T$.

Sol. Given matrix $A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 \\ 4 & 5 \end{bmatrix}$, then $A^T = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, $B^T = \begin{bmatrix} -3 & 4 \\ 5 & 5 \end{bmatrix}$.

$$AB = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -9+16 & 15+20 \\ -3+20 & 5+25 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 35 \\ 17 & 30 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 7 & 17 \\ 35 & 30 \end{bmatrix} \quad (1)$$

$$B^T A^T = \begin{bmatrix} -3 & 4 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -9+16 & -3+20 \\ 15+20 & 5+25 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 17 \\ 35 & 30 \end{bmatrix} \quad (2)$$

From eqn. (1) and (2)

$$(AB)^T = B^T A^T$$

Example 41. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, then find A^2.

Sol.

$$A^2 = A.A. = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+6 \\ 2+6 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}.$$

Example 42. If $A = \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$. Show that $A^2 - 5A + 5I = 0$, where I is unit matrix of order 2.

Sol. We have $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 9+1 & 3+2 \\ 3+2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 10 \end{bmatrix}$$

$$5I = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

LHS $A^2 - 5A + 5I$

$$\begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ 5 & 10 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10-15+5 & 5-5+0 \\ 5-5+0 & 5-10+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

EXERCISE-VI

1. Find the order of following matrices, also find their types

(a) $[1 \quad 5 \quad 7]$ (b) $\begin{bmatrix} 6 \\ 9 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & 1 \end{bmatrix}$ (f) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. If $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the value of x, y, z and w.

3. If $\begin{bmatrix} x+y & y-z \\ z-2x & y-x \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$, find x, y, z.

4. Construct a 2 × 2 matrix whose element $a_{ij} = \frac{(1+j)^2}{2}$.

5. If $A = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$, find A + 2B.

6. Find the value of x, y, z and a if $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$.

7. If A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$. Show that AB = BA = I.

8. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$. Find the number a and b such that $A^2 + aA + bI = 0$, where I is unit matrix of 2×2.

9. If A = $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$. Show that AB = BA.

10. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$, then evaluate AB + 2I.

11. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, then verify that

    (i) A(B + C) = AB + AC

    (ii) (B + C)A = BA + CA

12. If $A = \begin{bmatrix} 1 & 3 \\ x & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -3 \\ -2 & 3 \end{bmatrix}$. Verify that (A + B)C = AC + BC.

13. If A = $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $B = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then find AB.

14. If $X = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$. Find 3X + Y.

15. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, find :

    (i) 2A + 3B – 4I, when I is a unit matrix

    (ii) 3A – 2B

16. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, find 2A + 3B – 5I, where I is unit matrix.

17. Two matrices A_{mxn} & B_{pxq} can be multiplied only when
(a) m = p (b) n = p (c) m = q (d) n = q

18. If $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ then A^2 is
(a) $\begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ (d) None of these

ANSWERS

1. (a) order 1 × 3, Row matrix

   (b) order 3 × 1, Column matrix

   (c) order 2 × 2, Scalar matrix

   (d) order 2 × 2, Square matrix

   (e) order 2 × 3, Rectangular matrix

   (f) order 2 × 2, Null matrix

2. x = 2, y = 4, z = 1, and w = 3 3. x = 2, y = 1, z = 2

4. $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ 5. $\begin{bmatrix} -3 & 5 \\ 1 & 3 \end{bmatrix}$ 6. x = –3, y = –2, z = 4, a = 3

10. $\begin{bmatrix} 4 & 1 \\ 13 & 16 \end{bmatrix}$ 13. $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & b^2+a^2 \end{bmatrix}$

14. $\begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$ 15.(i) $\begin{bmatrix} 3 & 15 \\ 20 & 28 \end{bmatrix}$, (ii) $\begin{bmatrix} 4 & 3 \\ 4 & 9 \end{bmatrix}$ 16. $\begin{bmatrix} 2 & 15 \\ 2 & 24 \end{bmatrix}$

17. (b) 18. (a)

UNIT - 3

TRIGONOMETRY

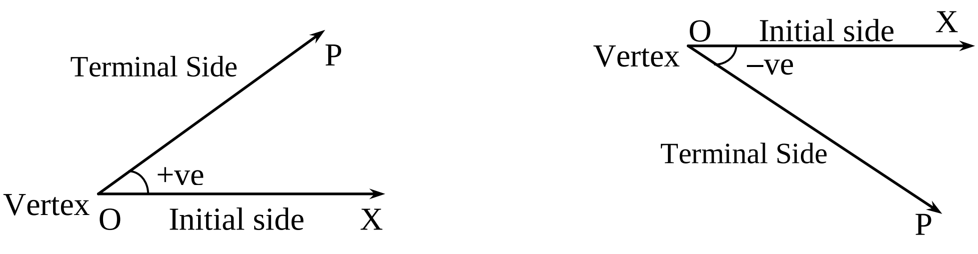
Learning Objectives

- Concept of angle: Measurement of angle in degrees, grades, radians and their conversions.
- T-Ratios of standard angle and fundamental identities, Allied angles (Without proof) Sum, Difference formulae and their applications, Product formulae (Transformation of product to sum, difference and vice versa,)
- Applications of Trigonometric terms in engineering problems such as to find an angle of elevation, height, distance etc.

Introduction: The word trigonometry is derived from two Greek words :trigono meaning 'a triangle' and metron meaning 'to measure'. Thus literally trigonometry means 'measurement of triangles'. In early stages of development of trigonometry, its scope lied in the measurement of sides and angles of triangles and the relationship between them. Though still trigonometry is largely used in that sense but of late it is also used in many other areas such as the science of seismology, designing electric circuits and many more areas.

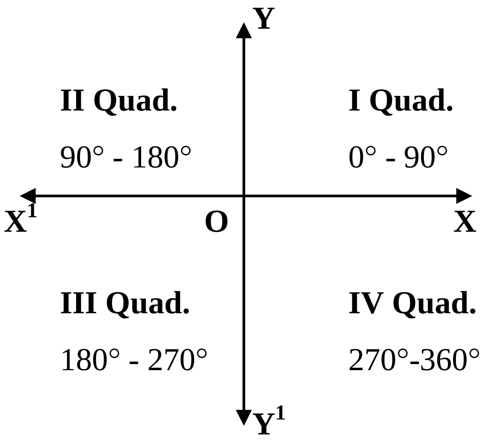
3.1 CONCEPT OF ANGLE

Definition: According to Euclid 'an angle is the inclination of a line to another line'. An angle may be of any magnitude and it may be positive or negative.



Angle in any quadrant

Two mutually perpendicular straight lines XOX' and YOY' divide the plane of paper into four parts XOY, YOX', X'OY and Y'OX' which are called I, II, III, IV quadrants respectively.



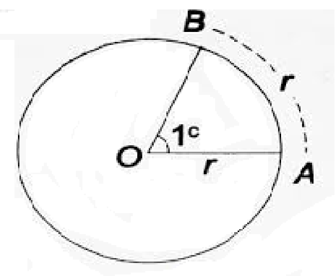
Measurement of Angle

Sometimes different units are used to measure the same quantity. For example, time is measured in hours, minutes and seconds. In the same manner, we shall now describe three most commonly used units of measurement of an angle.

(i) Sexagesimal OR the English system
(ii) Centesimal OR the French system
(iii) Circular measure system.

I	II	III
SexagesimalSystem	Centesimal System	Circular System
1rt∠ = 90°	1rt∠ = 100^g	Angle is measured in radians.
1° = 60′	1^g = 100′	Radian is an angle subtended
1′= 60″	1′= 100″	at the centre of a circle by an
		arc equal in length to the radius
		of that circle
		π^c = 180°
		or $\frac{\pi}{2}^{C}$ =1rt ∠

In the figure ∠ AOB = 1^c
So arc AB = OA = radius of the circle with centre O



Relation between three systems of an angle measurement

$$90^\circ = 100^g = \frac{\pi^c}{2}$$

RELATION BETWEEN DEGREES AND RADIANS

We have seen above that 1 radian $=\frac{2rt\angle s}{\pi}$

⇨ 1 radian $=\frac{180^{\circ}}{\pi}$ i.e. π radians = 180°

This relation enables us to express a radian in terms of degrees and a degree in terms of radians. The relation between degree measure and radian measure of some common angles are given in the following table:

Degree:	30	45	60	90	180	270	360
Radian:	π/6	π/4	π/3	π/2	π	3π/2	2π

Example 1. In which quadrant to the following angles lie?

(i) 790° (ii) -530°

Sol. (i) Dividing 790° by 360° (one complete revolution), we get

790° = 2 ×360° + 70°

Thus, the revolving line, after having made two complete revolution in the positive direction has further traced out an angle of 70° in the positive direction.

∴ the angle lies in 1^{st} quadrant.

(ii) Dividing -530° by 360° (one complete revolution), we get

-530° = -1×360° -170°

Thus, the revolving line, after having made one complete revolution in the negative direction has further traced out an angle of 170° in the negative direction.

∴ the angle lies in 3^{rd}quadrant.

Example 2. Write the following angles in circular measure

(i) 45° (ii) 75° (iii) 140° (iv) 18°20′

Sol.: (i) We know that 90° = $\frac{\pi^c}{2}$

$1° = \frac{\pi^c}{2} x \frac{1}{90}$

$45^0 = \frac{\pi^c}{2} x \frac{1}{90} \times 45 = \frac{\pi}{4}$

(ii) We know that

$90^0 = \frac{\pi^c}{2}$

$1^0 = \frac{\pi^c}{2} x \frac{1}{90}$

$75^0 = \frac{\pi^c}{2} x \frac{1}{90}\, x\, 75 = \frac{5\pi^c}{12}$

(iii) $90^0 = \frac{\pi^c}{2}$

$1^0 = \frac{\pi^c}{2} x \frac{1}{90}$

$140^0 = \frac{\pi^c}{2} x \frac{1}{90} x\ 140 = \frac{7\pi^c}{9}$

(iv) $18°20' = 18°+20'$

$= 18°+(20/60)°$

$= 18°+(1/3)°$

$= (18+1/3)° = 55/3°$

$=90^0 = \frac{\pi^c}{2}$

$1^0 = \frac{\pi^c}{2} x \frac{1}{90}$

$55/3° = \frac{\pi^c}{2} x \frac{1}{90} \times 55/3 = 11\pi/108$

Example 3. Find the centesimal measure of the angle whose radian measure are :

(i) $\frac{4\pi^c}{5}$ (ii) $\frac{3\pi^c}{10}$ (iii) $\frac{\pi}{20}$

Sol.(i) We know that

$$\frac{\pi^c}{2} = 100^g$$

$$\frac{\pi^c}{2} = 100$$

$$\pi = 200$$

$$\frac{4\pi}{5} = \frac{200}{5} \times 4 = 160^g$$

(ii)

$$\frac{\pi^c}{2} = 100^g$$

$$\frac{\pi^c}{2} = 100$$

$$\pi^c = 200$$

$$\frac{3\pi^c}{10} = 200\ x\ \frac{3}{10} = 60^g$$

(iii)

$$\frac{\pi^c}{2} = 100^g$$

$$\frac{\pi^c}{2} = 100$$

$\pi^c = 200$

$\pi^c/20 = 200^g/20 = 10^g$

Example 4. Write the following angles in sexagesimal measure whose radians measures are:

(i) $\frac{\pi^c}{5}$ (ii) $\frac{\pi^c}{6}$ (iii) $\pi^c/4$

Sol : (i) We know that

$$90° = \frac{\pi^c}{2}$$

$\pi^c = 180°$

$$\frac{\pi^c}{5} = 180^0 x \frac{1}{5} = 36^0$$

(ii) We know that

$\pi^c = 180°$

$$\frac{\pi^c}{6} = 180^0 x \frac{1}{6} = 30^0$$

(iii) We know that

$\pi^c = 180°$

$\pi^c/4 = 180°/4 = 45°$

EXERCISE- I

1. In which quadrant to the following angles lie

(i) 425° (ii) -540°

2. Express in radians the followings angles

(i) 135° (ii) 530° (iii) 40° 20'

3. Find the degree measures corresponding to the following radians measures

(i) $\left(\frac{\pi}{8}\right)^c$ (ii) $\left(\frac{7\pi}{12}\right)^c$ (iii) $\left(\frac{3\pi}{4}\right)^c$

4. Find Centesimal measure of the angle whose radian measure are

(i) $(5\pi/12)^c$ (ii) $(\pi/4)^c$

ANSWERS

1. (i) 1^{st} Quad. (ii) 2^{nd} Quad.

2. (i) $3\pi/4$ radians (ii) $\frac{53\pi}{18}$ radians (iii) $\frac{121\pi}{540}$ radians

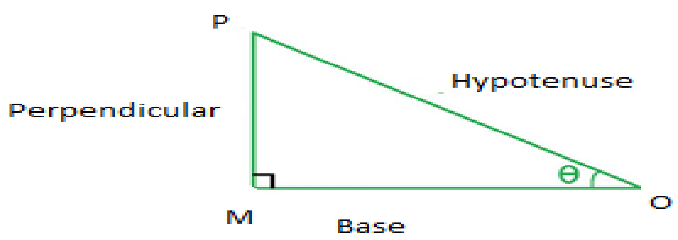
3. (i) 22°30' (ii) 105° (iii) 42° 57' 17"

4. (i) $(250/3)^g$ (ii) 50^g

3.2 TRIGONOMETRIC RATIOS OF ANGLES

Trigonometric ratios are used to find the remaining sides and angles of triangles, when some of its sides and angles are given. This problem is solved by using some ratios of sides of a triangle with respect to its acute angles. These ratios of acute angles are called trigonometric ratios.

Trigonometric Ratios: In right angled triangle OPM



(i) $\frac{MP}{OP}\frac{(Perpendicular)}{Hypotenuse}$ is called the sine of angle θ and written as sin θ.

(ii) $\frac{OM}{OP}\left(\frac{Base}{Hypotenuse}\right)$ is called the cosine of angle θ and written as cosθ.

(iii) $\frac{MP}{OM}\left(\frac{Perpendicular}{Base}\right)$ is called the tangent of angle θ and written as tan θ.

(iv) $\frac{OM}{MP}\left(\frac{\text{Base}}{\text{Perpendicular}}\right)$ is called cotangent of angle θ and written as cot θ.

(v) $\frac{OP}{OM}\left(\frac{\text{Hypotenuse}}{\text{Base}}\right)$ is called secant of angle θ and written as sec θ.

(vi) $\frac{OP}{MP}\left(\frac{\text{Hypotenus}}{\text{Perpendicular}}\right)$ is called cosecant of angle θ and written as cosec θ.

Relation between trigonometric ratios

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

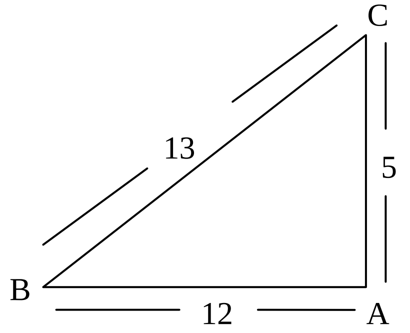
(ii) $\sin^2\theta + \cos^2\theta = 1$

(iii) $\sec^2\theta - \tan^2\theta = 1$

(iv) $\text{cosec}^2\theta - \cot^2\theta = 1$

Example 5. In a ΔABC, right angle at A, if AB = 12, AC = 5 and BC = 13. Find the value of sin B, cos B and tan B.

Sol. In right angled ΔABC ;



Base = AB = 12,

Perpendicular = AC = 5

Hypotenus = BC = 13

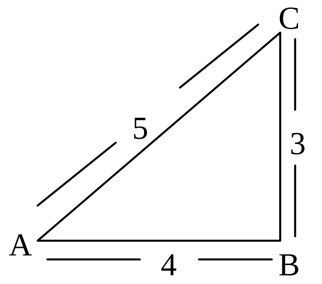
$\sin B = \frac{AC}{BC} = \frac{5}{13}$

$\cos B = \frac{AB}{BC} = \frac{12}{13}$

$\tan B = \frac{AC}{AB} = \frac{5}{12}$

Example 6. In a ΔABC, right angled at B if AB = 4, BC = 3, find the value of sin A and cos A.

Sol : We know by Pythagoras theorem



$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 4^2 + 3^2$$

$$= 16 + 9 = 25$$

$$AC^2 = (5)^2 \quad \therefore AC = 5$$

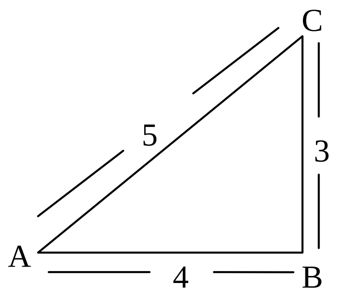
$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{3}{5}$$

$$\cos A = \frac{AB}{AC} = \frac{4}{5}.$$

Example 7. If $\sin A = \frac{3}{5}$ find the value of cos A and tan A.

Sol : We know that $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$.

By Pythagoras theorem



$$AC^2 = AB^2 + BC^2$$

$$5^2 = AB^2 + 3^2$$

$$25 = AB^2 + 9$$

$$AB^2 = 25 - 9 = 16$$

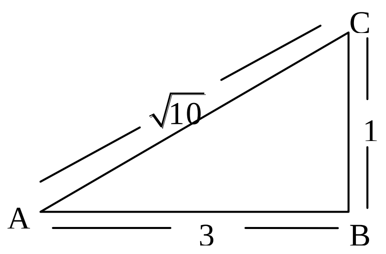
$$AB = 4$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

Example 8. If cosec A = $\sqrt{10}$. Find the values of sin A, cos A.

Sol : We have $\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$



In a right angled triangle ABC.By Pythagoras theorem.

$AC^2 = AB^2 + BC^2$

$(\sqrt{10})^2 = AB^2 + (1)^2$

$10 = AB^2 + 1$

$AB^2 = 9 \qquad \therefore\ AB = 3$

$\therefore \qquad \sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{1}{\sqrt{10}}$

$\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{3}{\sqrt{10}}$

EXERCISE-II

1. In a right triangle ABC, right angle at B, if $\sin A = \frac{3}{5}$, find the value of cos A and tan A.

2. In a ΔABC, right angle at B, if AB = 12, BC = 5, find sin A and tan A.

3. In a ΔABC, right angled at B, AB = 24cm, BC 7cm. Find the value of sin A, cos A

4. If $\tan\theta = \frac{3}{5}$, find the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$.

5. If $3\tan\theta = 4$, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$.

6. If $\cot\theta = \frac{7}{8}$. Find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

ANSWERS

1. $\cos A = \frac{4}{5}, \tan A = \frac{3}{4}$ 2. $\sin A = \frac{5}{13}, \tan A = \frac{5}{12}$ 3. $\frac{7}{25}, \frac{24}{25}$

4. 4 5. $\frac{4}{5}$ 6. $\frac{49}{64}$.

T-Ratios of Standard Angles

The angles 0°, 30°, 60°, 90°, 180°, 270° and 360° are called standard angles. The value of 0°, 30°, 45°, 60° and 90° can be remembered easily with the help of following table:

Table

Angleθ	0°	30°	45°	60°	90°
$\sin\theta$	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}}=1$
$\cos\theta$	$\sqrt{\frac{4}{4}}=1$	$\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{0}{4}}=0$
$\tan\theta$	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{2}}=1$	$\sqrt{\frac{3}{1}}=\sqrt{3}$	$\sqrt{\frac{4}{0}}=\infty$

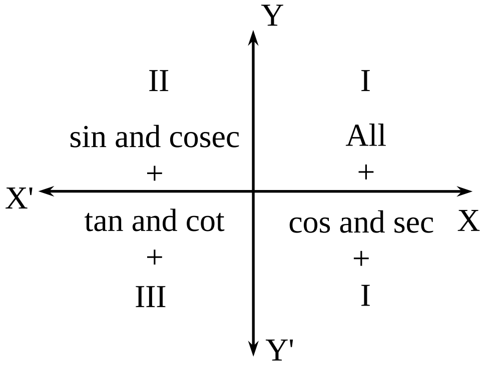
Signs of Trigonometric Ratios: The sign of various t-ratios in different quadrants are

(i) In first quadrant all the six t-ratios are positive.

(ii) In second quadrant only sin θ and cosec θ are positive and remaining t-ratios are negative.

(iii) In third quadrant only tan θ and cot θ are positive and remaining t-ratios are negative.

(iv) In fourth quadrant cosθ and sec θ are positive and remaining t-ratios are negative.



T-ratios of Allied angles :

Allied angles : Two anglesare saidto be allied angles when their sum or differences is either zero or a multiple of 90°.

Complimentary angles : Two angles whose sum is 90° are called complement of each other. The angle θ and 90°–θ are complementary of each other.

Supplementary angles : Two angles whose sum is 180° are called supplementary of each other. The angles θ and 180°–θ are supplementary of each other.

The value of –θ, 90 ±θ, 180 ±θ, 270 ±θ and 360 ±θ can be remember easily by the following table.

Angle	**Sin**	**Cos**	**Tan**	**Remarks**
–θ	sin(–θ) = –sin θ	cos(–θ) = cosθ	tan (–θ) = tanθ	
90° –θ	cosθ	sin θ	cot θ	Co–formulae apply
90° + θ	cosθ	–sin θ	–cot θ	Co–formulae apply
180° –θ	sin θ	–cosθ	– tan θ	
180° + θ	–sin θ	–cosθ	tan θ	
270° –θ	–cosθ	– sin θ	cot θ	Co–formulae apply
270° + θ	–cosθ	sin θ	–cot θ	Co–formulae apply
360° –θ	–sin θ	cosθ	–tan θ	
360° + θ	sin θ	cosθ	tan θ	

Example 9. Find the value of:

(i) sin 135° (ii) sin 300°.

Sol : (i) sin 135° = sin (90° + 45°) ∵ sin(90° + **θ**) = cos**θ**

$$= \cos 45° = \frac{1}{\sqrt{2}}$$

OR

sin 135° = sin (180°– 45°)

$$= \sin 45° = \frac{1}{\sqrt{2}}$$

(ii) $\sin 300° = \sin(360° - 60°)$ $\because \sin(360° - \boldsymbol{\theta}) = -\sin \boldsymbol{\theta}$

$$= -\sin 60° = \frac{-\sqrt{3}}{2}$$

Example 10. Evaluate

(i) tan 120° (ii) sin 150° (iii) cos 300° (iv) cot 225°

(v) sin(–690°)

Sol : (i) $\tan 120° = \tan(90° + 30°) = -\cot 30° = -\sqrt{3}$

(ii) $\sin 150° = \sin(180° - 30°) = \sin 30° = \frac{1}{2}$

(iii) $\cos 300° = \cos(360° - 60°) = \cos 60° = \frac{1}{2}$

(iv) $\cot 225° = \cot(180° + 45°) = \cot 45° = 1$

(v) $\sin(-690°) = -\sin 690° \quad = -\sin(7 \times 90° + 60°)$

$$= \cos 60° \qquad = +\cos 60° \qquad = \frac{1}{2}$$

Example 11. Evaluate

(i) cos (–750°) (ii) sin (–240°) (iii) sin 765° (iv) cos 1050°

(v) tan (–1575°)

Sol : (i) $\cos(-750°) = +\cos 750°$

$$= \cos(2 \times 360° + 30°)$$

$$= \cos 30° = \frac{\sqrt{3}}{2}$$

(ii) $\sin(-240°) = -\sin 240°$

$$= -\sin(180° + 60°)$$

$$= \sin 60° = -\frac{\sqrt{3}}{2}$$

(iii) $\sin 765° = \sin(2 \times 360° + 45°)$

$= \sin 45° = \frac{1}{\sqrt{2}}$

(iv) $\cos(1050°) = \cos(3 \times 360° - 30°)$

$= \cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

(v) $\tan(-1575°) = -\tan 1575°$

$= -\tan(4 \times 360° + 135°)$

$= -\tan 135°$

$= -\tan(180° - 45°)$

$= \tan 45° = 1$

Ex. 12. Evaluate the following :(i) $\frac{\cos 37°}{\sin 53°}$ (ii) $\sin 39° - \cos 51°$

Sol :(i) $\frac{\cos 37°}{\sin 53°} = \frac{\cos(90° - 53°)}{\sin 53°} = \frac{\sin 53°}{\sin 53°} = 1$

(ii) $\sin 39° - \cos 51°$

$= \sin(90° - 51°) - \cos 51°$

$= \cos 51° - \cos 51° = 0.$

EXERCISE-III

1. Evaluate the following :

(i) $\tan 225°$ (ii) $\sin 315°$ (iii) $\cos 150°$

(iv) $\frac{\sin 41°}{\cos 49°}$ (v) $\frac{\tan 54°}{\cot 36°}$

2. Evaluate the following :

(i) Cosec (-210°) (ii) Cot (-120°)

3. Find the value of :

(i) sin 300° (ii) tan240° (iii) cos330°

ANSWERS

1. (i) 1 (ii) $-1/\sqrt{2}$ (iii) $-\sqrt{3}/2$ (iv) 1 (v) 1

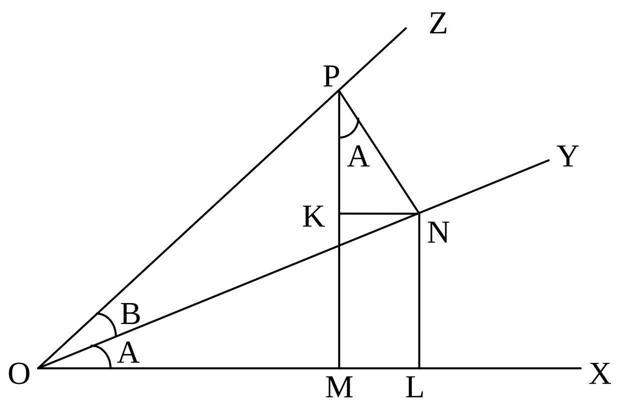
2. (i) 2 (ii) $1/\sqrt{3}$

3. (i) $-\dfrac{\sqrt{3}}{2}$ (ii) $\sqrt{3}$ (iii) $\sqrt{3}/2$

Addition and Subtraction Formulae

A. <u>Addition Formulae:</u>

Let a revolving line starting from OX, trace out an angle $\angle XOY = A$ and let it revolve further to trace an angle $\angle YOZ = B$. So that $\angle XOZ = A + B$ (Addition of angles A and B).



(1) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(2) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

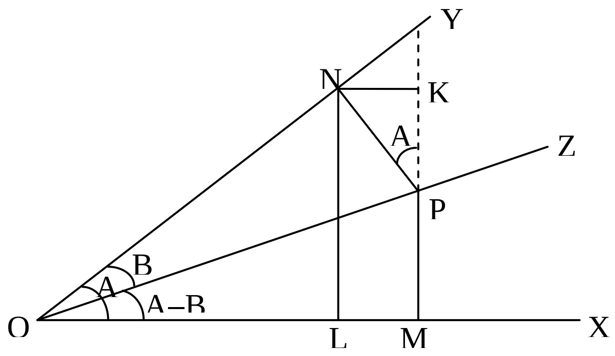
(3) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(4) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(5) $\tan(45 + A) = \dfrac{1 + \tan A}{1 - \tan A}$

B. <u>Subtraction formulae</u>:

Let a revolving line, starting from OX, trace out an angle $\angle XOY = A$ and let it revolve back to trace an angle $\angle YOZ = B$. So that $\angle XOZ = A - B$(Subtraction of Angle A and B)



(1) sin (A – B) = sin A cos B –cos A sin B

(2) cos (A – B) = cos A cos B + sin A sin B

(3) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(4) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(5) $\tan(45° - A) = \dfrac{1 - \tan A}{1 + \tan A}$

Example 13. Evaluate

(i) sin 15°, cos 15°, tan 15° (ii) sin 75°, cos 75°, tan 75°

Sol : (i) (a) sin 15° = sin (45°– 30°)

= sin 45°cos 30°–cos 45° sin 30° [sin(A – B) = sin A cos B –cos A sin B]

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(b) cos 15° = cos (60°– 45°) [cos(A – B) = cosAcosB + sinAsinB]

= cos 60°cos 45° + sin 60° sin 45°

$$= \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{1}{4}(\sqrt{2}+\sqrt{6})$$

(c) tan 15° = tan (45°– 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

(ii) (a) sin 75° = sin (45° + 30°)

= sin 45°cos 30° + cos 45° sin 30° $[\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

(b) cos 75° = cos (45° + 30°)

= cos 45°cos 30°– sin 45° sin 30° [cos (A + B) = cos A cos B – sin A sin B]

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(c) tan 75° = tan (45° + 30°)

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45°.\tan 30°} \qquad \left[\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Example 14. Write down the values of :

(i) cos 68°cos 8° + sin 68° sin 8° (ii) cos 50°cos 10°– sin 50° sin 10°

Sol :(i) cos 68°cos 8° + sin 68° sin 8°

= cos (68°– 8°) [cos(A – B) = cos A cos B + sin A sin B]

$= \cos 60° = \frac{1}{2}$

(ii) cos 50°cos 10°– sin 50° sin 10°

= cos (50° + 10°) [cos(A + B) = cos A cos B – sin A sin B]

$= \cos 60° = \frac{1}{2}$

Example 15. Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$

Sol : L.H.S. = $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

$$= \frac{1 + \frac{\sin 11°}{\cos 11°}}{1 - \frac{\sin 11°}{\cos 11°}}$$ [Dividing the num. and denom. by cos 11°]

$$= \frac{1 + \tan 11°}{1 - \tan 11°} = \tan(45° + 11°) \qquad \left[\tan(45^0 + A) = \frac{1 + \tan A}{1 - \tan A}\right]$$

= tan 56° = R.H.S.

Example 16. Prove that tan3A tan2A tanA = tan3A – tan2A –tanA.

Sol : We can write, tan 3A = tan (2A + A)

$$\Rightarrow \qquad \tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$\Rightarrow$ tan 3A – tan 3A tan 2A tan A = tan 2A + tan A

$\Rightarrow$ tan 3A tan 2A tan A = tan 3A – tan 2A – tan A

Example 17. If tan A = $\sqrt{3}$, tan B = 2 – $\sqrt{3}$, find the value of tan (A – B).

Sol : Using the formula; $\tan(A-B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}=\dfrac{\sqrt{3}+2-\sqrt{3}}{1-\sqrt{3}(2-\sqrt{3})}=\dfrac{2}{1-2\sqrt{3}+3}$

$$=\frac{2}{4-2\sqrt{3}}=\frac{2}{2(2-\sqrt{3})}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$$

$\therefore$ tan(A – B) = $2+\sqrt{3}$

Example 18. If A and B are acute angles and sin A = $\dfrac{1}{\sqrt{10}}$, $\sin B=\dfrac{1}{\sqrt{5}}$. Prove that A+B = $\dfrac{\pi}{4}$.

Sol : Given, sin A = $\dfrac{1}{\sqrt{10}}$ and $\sin B=\dfrac{1}{\sqrt{5}}$

We know, cos A = $\sqrt{1-\sin^2 A}$ and $\cos B=\sqrt{1-\sin^2 B}$ [$\because$ A and B are acute angles]

$\Rightarrow$ cos A = $\sqrt{1-\dfrac{1}{10}}$ and cos B = $\sqrt{1-\dfrac{1}{5}}$

$\Rightarrow$ cos A = $\sqrt{\dfrac{9}{10}}$ and cos B = $\sqrt{\dfrac{4}{5}}$

$\Rightarrow$ cos A = $\dfrac{3}{\sqrt{10}}$ and cos B = $\dfrac{2}{\sqrt{5}}$

Now cos (A + B) = cos A cos B – sin A sin B

$$=\frac{3}{\sqrt{10}}.\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{10}}.\frac{1}{\sqrt{5}}=\frac{6}{\sqrt{50}}-\frac{1}{\sqrt{50}}$$

$$=\frac{5}{\sqrt{50}}=\frac{5}{5\sqrt{2}}=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}$$

Hence A + B = $\dfrac{\pi}{4}$

Example 19. Prove that tan 70° = tan 20° + 2 tan 50°.

Sol : We can write; tan 70° = tan (20° + 50°)

$$\Rightarrow \quad \tan 70° = \frac{\tan 20° + \tan 50°}{1 - \tan 20° \tan 50°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$\Rightarrow$ tan 70°– tan 20° tan 50° tan 70° = tan 20° + tan 50°

$\Rightarrow$ tan 70°– tan 20° tan 50° tan (90°– 20°) = tan 20° + tan 50°

$\Rightarrow$ tan 70°– tan 20° tan 50° cot 20° = tan 20° + tan 50°

$\Rightarrow$ tan 70°– tan 50° = tan 20° + tan 50° [∵ tan □. cot □ = 1]

$\Rightarrow$ tan 70° = tan 20° + 2 tan 50°

Example 20. Prove that tan 13 A – tan 9A – tan 4A = tan 13A tan 9A tan 4A.

Sol : We can write; tan 13A = tan (9A + 4A)

$$\tan 13A = \frac{\tan 9A + \tan 4A}{1 - \tan 9A \tan 4A} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$\Rightarrow$ tan 13A – tan 13A tan 9A tan 4A = tan 9A + tan 4A

$\Rightarrow$ tan 13A – tan 9A – tan 4A = tan 13A tan 9A tan 4A

Example 21. Prove that $\tan 2\theta - \tan\theta = \tan\theta \sec 2\theta$

Sol : L.H.S. = $\tan 2\theta - \tan\theta$

$$= \frac{\sin 2\theta}{\cos 2\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\sin 2\theta \cos\theta - \cos 2\theta \sin\theta}{\cos 2\theta \cos\theta}$$

$$= \frac{\sin(2\theta - \theta)}{\cos 2\theta \cos\theta} = \frac{\sin\theta}{\cos 2\theta \cos\theta} \qquad [\sin(A - B) = \sin A \cos B - \cos A \cos B]$$

$$= \frac{\sin\theta}{\cos\theta \cos 2\theta} = \tan\theta \sec 2\theta.$$

Example 22. Proveby using trigonometric formulae that; tan 65° = tan 25° + 2 tan 40°.

Sol : We can write; 65° = 40° + 25°

$$\tan 65° = \tan(40° + 25°) = \frac{\tan 40° + \tan 25°}{1 - \tan 40° \tan 25°}$$

$$\tan 65° - \tan 40° \tan 25° \tan 65° = \tan 40° + \tan 25°$$

$$\tan 65° - \tan 40° \tan 25° \tan(90° - 25°) = \tan 40° + \tan 25°$$

$$\tan 65° - \tan 40° \tan 25° \cot 25° = \tan 40° + \tan 25°$$

$$\tan 65° - \tan 40° = \tan 40° + \tan 25° \qquad [\because \tan \theta \cot \theta = 1]$$

$$\tan 65° = \tan 25° + 2 \tan 40°$$

Hence proved

EXERCISE– IV

1. Evaluate (i) sin 105° (ii) cos 105° (iii) tan 105°.

2. Evaluate : (i) sin 22°cos 38° + cos 22° sin 38° (ii) $\frac{\tan 66° + \tan 69°}{1 - \tan 66° \tan 69°}$

   (iii) sin50° cos20°-cos50° sin20° (iv) cos70° cos10°+sin70°sin10°

3. Prove that :

   (i) sin 105° + cos 105° = cos 45°

   (ii) cos A =$\frac{4}{5}$ and cos B = $\frac{3}{5}$, where 0< A <$\frac{\pi}{2}$; 0<B<$\frac{\pi}{2}$ find sin (A + B) and cos (A + B)

   (iii) If tan A = $\frac{5}{6}$ and tan B = $\frac{1}{11}$. Show that A + B = $\frac{\pi}{4}$.

4. Prove that :

   (i) $\tan 28° = \frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$

   (ii) $\tan 58° = \frac{\cos 13° + \sin 13°}{\cos 13° - \sin 13°}$

5. If cos A = $\frac{1}{7}$ and cos B = $\frac{13}{14}$. Prove that A – B = 60°. A and B are acute angles.

6. Prove that :

   (i) tan 55° = tan 35° + 2 tan 20°

   (ii) tan 50° = tan 40° + 2 tan 10°

   (iii) 2 tan 70° = tan 80°– tan 10°

7. If tan A = $\frac{1}{2}$ and tan B = $\frac{1}{3}$. Show that A + B = 45°. Given that A and B are positive acute angles.

8. Show that tan9A-tan5A-tan4A= tan4Atan5Atan9A

9. Prove that $\sqrt{3}\cos 23° - \sin 23° = 2\cos 53°$.

10. Show that tan3θ-tan2θ-tanθ = tan3θ tan2θ tanθ.

ANSWERS

1. (i) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (ii) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (iii) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

2. (i) $\frac{\sqrt{3}}{2}$ (ii) 1 (iii) $\frac{1}{2}$ (iv) $\frac{1}{2}$

3. (i) $\frac{220}{221}, \frac{220}{221}$

Product formulae (Transformation of a Product into a Sum or Difference)

(i) 2 sin A cos B = sin (A + B) + sin (A – B)

(ii) 2 cos A sin B = sin (A + B) – sin (A – B)

(iii) 2 cos A cos B = cos (A + B) + cos (A – B)

(iv) 2 sin A sin B = cos (A – B) –cos (A + B)

Aid to memory

2 sin A cos B = sin (sum) + sin (difference)

2 cos A sin B = sin (sum) – sin (difference)

2 cos A cos B = cos (sum) + cos (difference)

2 sin A sin B = cos (difference) –cos (sum)

Example 23. Express the following as a sum or difference

(i) 2 sin 5x cos 3x (ii) 2 sin 4x sin 3x (iii) 8 cos 8x cos 4x

Sol :(i) 2 sin 5x cos 3x = sin (5x + 3x) + sin (5x – 3x)

$= \sin 8x + \sin 2x$

(ii) $2 \sin 4x \sin 3x = \cos (4x - 3x) - \cos (4x + 3x) = \cos x - \cos 7x$.

(iii) $8 \cos 8x \cos 4x = 4[2 \cos 8x \cos 4x] = 4[\cos (8x + 4x) + \cos (8x - 4x)]$

$= 4[\cos 12x + \cos 4x]$

Example 24. Prove that $\cos 20°\cos 40°\cos 60°\cos 80° = \frac{1}{16}$.

Sol : L.H.S. $= \cos 20°\cos 40° \frac{1}{2} \cos 80°$

$$= \frac{1}{2} (\cos 20°\cos 40°) \cos 80°$$

$$= \frac{1}{4} (2 \cos 20°\cos 40°) \cos 80°$$

$$= \frac{1}{4} (\cos 60° + \cos 20°) \cos 80°$$

$$= \frac{1}{4}\left[\frac{1}{2} + \cos 20^0\right]\cos 80^O$$

$$= \frac{1}{4}\left[\frac{1}{2}\cos 80° + \cos 20° \cos 80°\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}\cos 80° + \frac{1}{2}(2\cos 20° \cos 80°)\right]$$

$$= \frac{1}{8} [\cos 80° + \cos 100° + \cos 60°]$$

As $\cos 100° = \cos (180° - 80°) = -\cos 80°$

$$= \frac{1}{8} [\cos 80° - \cos 80° + \frac{1}{2}] = \frac{1}{16} = \text{ R.H.S.}$$

Example 25. Prove that, $\cos 10°\cos 50°\cos 70° = \frac{\sqrt{3}}{8}$.

Sol : LHS $= \frac{1}{2} [\cos 10°(2\cos 50°\cos 70°)]$

$$= \frac{1}{2} [\cos 10°(\cos 120^o + \cos 20°)]$$

$= \frac{1}{2}$ [cos 10°($-\frac{1}{2}$ + cos 20°)]

$= -\frac{1}{4}\cos 10° + \frac{1}{4}(2\cos 10° \cos 20°)$

$= -\frac{1}{4}\cos 10° + \frac{1}{4}(\cos 30° + \cos 10°)$

$= -\frac{1}{4}\cos 10° + \frac{1}{4}\cos 30° + \frac{1}{4}\cos 10°$

$= \frac{1}{4}\cos 30° = \frac{1}{4}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$ = R.H.S

Example 26. Prove that sin 20° sin 40° sin 60° sin 80°= $\frac{3}{16}$.

Sol : L.H.S $= \frac{\sqrt{3}}{2}\sin 20° \sin 40° \sin 80°$ [$\sin 60^{O} = \frac{\sqrt{3}}{2}$]

$= \frac{1}{2}.\frac{\sqrt{3}}{2}\sin 20°(2\sin 80° \sin 40°)$

$= \frac{\sqrt{3}}{4}\sin 20°(\cos 40° - \cos 120°)$ ∵2sinAsinB=cos(A–B)–cos(A+B)

$= \frac{\sqrt{3}}{8}(2\sin 20° \cos 40°) + \frac{\sqrt{3}}{8}\sin 20°$

$= \frac{\sqrt{3}}{8}(\sin 60° - \sin 20°) + \frac{\sqrt{3}}{8}\sin 20°$

$= \frac{\sqrt{3}}{8}\sin 60° - \frac{\sqrt{3}}{8}\sin 20° + \frac{\sqrt{3}}{8}\sin 20°$

$= \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} = \frac{3}{16}$ = R.H.S.

Transformation of a sum or difference into a product formulae

(i) sin C + sin D = 2 sin $\frac{C+D}{2}\cos\frac{C-D}{2}$

(ii) sin C – sin D = $2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(iii) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(iv) $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

Example 27. Express the following as product :

(i) $\sin 14x + \sin 2x$ (ii) $\cos 10° - \cos 50°$ (iii) $\sin 80° - \sin 20°$

Sol :(i) $\sin 14x + \sin 2x = 2\sin\frac{14x+2x}{2}\cos\frac{14x-2x}{2} = 2\sin 8x\cos 6x$.

(ii) $\cos 10° - \cos 50° = 2\sin\frac{10°+50°}{2}\sin\frac{50°-10°}{2} = 2\sin 30°\sin 20°$

(iii) $\sin 80° - \sin 20° = 2\cos\frac{80°+20°}{2}\sin\frac{80°-20°}{2} = 2\cos 50°\sin 30°$

Example 28. Prove that

(i) $\frac{\sin A + \sin B}{\cos A + \cos B} = \tan\frac{A+B}{2}$, (ii) $\frac{\cos 8x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$

Sol : (i) L.H.S. $= \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}$

$$= \frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}} = \tan\left(\frac{A+B}{2}\right) = \text{R.H.S.}$$

(ii) L.H.S. $= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{2\sin\frac{9x+5x}{2}\sin\frac{5x-9x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$$= \frac{-2\sin 7x\sin 2x}{2\cos 10x\sin 7x} = \frac{-\sin 2x}{\cos 10x} = \text{R.H.S.}$$

Example 29. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$.

Sol : $\frac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$

$$= \frac{2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2}+\cos 3x}{2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2}+\sin 3x}$$

$$= \frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$= \frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)} = \frac{\cos 3x}{\sin 3x} = \cot 3x .$$

Example 30. Prove that

(i) $\frac{\sin A+\sin B}{\cos A+\cos B} = \tan\frac{A+B}{2}$ (ii) $\frac{\sin 7A+\sin 3A}{\cos 7A+\cos 3A} = \tan 5A$.

Sol :(i) L.H.S = $\frac{\sin A+\sin B}{\cos A+\cos B}$

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \tan\frac{A+B}{2} = \text{R.H.S.}$$

(ii) L.H.S = $\frac{\sin 7A+\sin 3A}{\cos 7A+\cos 3A}$

$$= \frac{2\sin\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}{2\cos\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}$$ [Using CD formula]

$$= \frac{\sin 5A\cos 2A}{\cos 5A\cos 2A} = \tan 5A = \text{R.H.S.}$$

Example 31. Prove that

(i) $\sin 47° + \cos 77° = \cos 17°$ (ii) $\sin 51° + \cos 81° = \cos 21°$

(iii) $\cos 52° + \cos 68° + \cos 172° = \cos 20° + \cos 100° + \cos 140°$

Sol : L.H.S = $\sin 47° + \cos 77°$

$= \sin 47° + \cos (90° - 13°) = \sin 47° + \sin 13°$

$= 2\sin\frac{47°+13°}{2}\cos\frac{47°-13°}{2}$ $[\because \sin C+\sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}]$

$= 2\sin 30°\cos 17° = \frac{2}{2}\cos 17° = \cos 17° = \text{RHS}$ $[\because \sin 30 = \frac{1}{2}]$

(ii) L.H.S. = $\sin 51° + \cos 81° = \sin (90° - 39°) + \cos 81°$

$= \cos 39° + \cos 81°$ [$\because$ sin (90°–□) = cos□]

$= 2\cos\frac{81°+39°}{2}\cos\frac{81°-39°}{2}\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2 \cos 60° \cos 21°$

$= 2 \times \frac{1}{2} \cos 21° = \cos 21°$ R.H.S. [$\because \cos 60° = \frac{1}{2}$]

L.H.S $= \cos 52° + \cos 68° + \cos 172°$

$= 2\cos\frac{52°+68°}{2}\cos\frac{68°-52°}{2} + \cos 172°$

$= 2 \cos 60° \cos 8° + \cos 172°$

$= 2 \times \frac{1}{2} \cos 8° + \cos (180° - 8°)$

$= \cos 8° - \cos 8° = 0$ [$\because \cos(180° - \theta) = -\cos\theta$]

R.H.S $= \cos 20° + \cos 100° + \cos 140°$

$= 2\cos\frac{100°+20°}{2}\cos\frac{100°-20°}{2} + \cos 140°$

$= 2 \cos 60° \cos 40° + \cos 140°$

$= 2 \times \frac{1}{2} \cos 40° + \cos (180° - 40°)$

$= \cos 40° - \cos 40° = 0$ [$\because \cos(180° - \theta) = -\cos\theta$]

$\therefore$ L.H.S = R.H.S

Example 32. $\cos A + \cos(120° - A) + \cos(120° + A) = 0$

Sol: L.H.S $= \cos A + \cos (120° - A) + \cos (120° + A)$

$= \cos A + 2\cos\left(\frac{120° + A + 120° - A}{2}\right)\cos\left(\frac{120° + A - 120° + A}{2}\right)$

$= \cos A + 2 \cos 120° \cos A = \cos A + 2\left(-\frac{1}{2}\right)\cos A$

$= \cos A - \cos A = 0 =$ R.H.S

EXERCISE- V

1. Express as sum or difference:

   (i) $2 \sin 4\theta \cos 2\theta$ (ii) $2 \sin\theta \cos 3\theta$

2. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$.

3. Prove that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$.

4. Prove that $\sin 10° \sin 50° \sin 60° \sin 70° = \frac{\sqrt{3}}{16}$.

5. Express the following as a product :

   (i) $\sin 7\theta + \sin 3\theta$ (ii) $\cos 5\theta + \cos 3\theta$

   (iii) $\sin 5\theta - \sin \theta$ (iv) $\cos 2\theta - \cos 4\theta$

6. Prove that : (i) $\frac{\cos A - \cos 3A}{\sin 3A - \sin A} = \tan 2A$ (ii) $\frac{\sin 7x + \sin 3x}{\cos 7x + \cos 3x} = \tan 5x$

7. Prove that $\cos 28° - \sin 58° = \sin 2°$.

8. Prove that :

   (i) $\cos 52° = \cos 68° + \cos 172° = 0$

   (ii) $\sin 50° - \sin 70° = \sin 10° = 0$

9. Prove that $\sqrt{3} \cos 13° + \sin 13° = 2 \sin 13°$.

10. Prove that $\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$.

<u>**ANSWERS**</u>

1. (i) $\sin 6\theta + \sin 2\theta$ (ii) $\sin 4\theta - \sin 2\theta$

5. (i) $2 \sin 5\theta \cos 2\theta$ (ii) $2 \cos 3\theta \sin 2$ (iii) $2 \cos 4\theta \cos\theta$ (iv) $2 \sin 3\theta \sin \theta$

T–Ratios of Multiple and Submultiple Angles

(i) $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$.

(iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

Remember

(i) sin(any angle) = 2 sin (half angle) cos (half angle)

(ii) cos (any angle) = $\cos^2$ (half angle) – $\sin^2$ (half angle)

= 2 $\cos^2$ (half angle) – 1

= 1 – 2 $\sin^2$ (half angle)

$= \dfrac{1-\tan^2(\text{half angle})}{1+\tan^2(\text{half angle})}$

(iii) tan(any angle) = $\dfrac{2\tan(\text{half angle})}{1-\tan^2(\text{half angle})}$

Remember

(i) $\sin^2$(any angle) = $\dfrac{1-\cos(\text{double the angles})}{2}$

(ii) $\cos^2$ (any angle) = $\dfrac{1+\cos(\text{double the angles})}{2}$

(iii) $\tan^2$ (any angle) = $\dfrac{1-\cos(\text{double the angles})}{1+\cos(\text{double the angles})}$

T–ratios of 3A in terms of those of A

(i) $\sin 3A = 3\sin A - 4\sin^3 A$

(ii) $\cos 3A = 4\cos^3 A - 3\cos A$

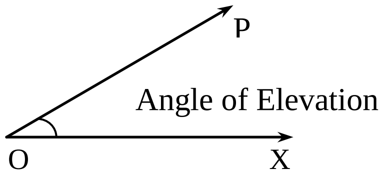
(iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

3.3 APPLICATIONS OF TRIGONOMETRIC TERMS IN ENGINEERING PROBLEMS

Height and Distance: Trigonometry helps to find the height of objects and the distance between points.

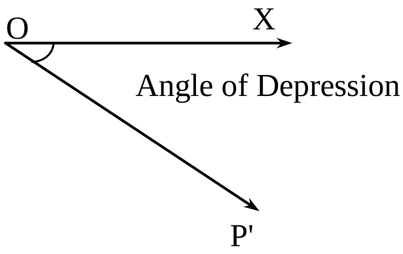
Angle of Elevation : The angle of elevation is for objects that are at a level higher than that of the observer.

$\angle XOP$ is called angle of elevation.



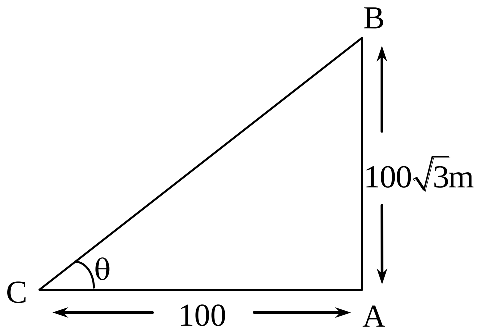
Angle of Depression : The angle of depression is for objects that are at a level which is lower than that of the observer.

$\angle XOP'$ is called angle of depression.



Example 33. A tower is $100\sqrt{3}$ metres high. Find the angle of elevation of its top from a point 100 metres away from its foot.

Sol : Let AB the tower of height $100\sqrt{3}$ m and let C be a point at a distance of 100 metres from the foot of tower. Let θ be the angle of elevation of the top of the tower from point C. In right angle ΔCAB



$$\frac{AB}{AC} = \tan\theta$$

$$\frac{100\sqrt{3}}{100} = \tan\theta \qquad \therefore$$

$$\tan\theta = \sqrt{3} = \tan 60°$$

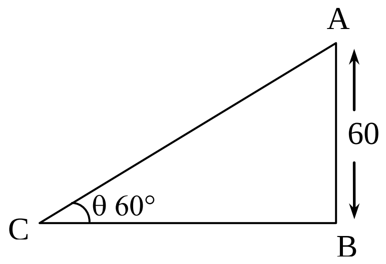
$$\Rightarrow \qquad \theta = 60°,$$

Hence the angle of elevation of the top of the tower from a point 100 metre away from its foot is 60°.

Example 34. A kite is flying at a height of 60 metres above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Sol : Let A be the kite and CA be the string attached to the kite such that its one end is tied to a point C on the ground. The inclination of the string CA with the ground is 60°.

` In right angle ΔABC we have



$$\frac{AB}{AC} = \sin\theta$$

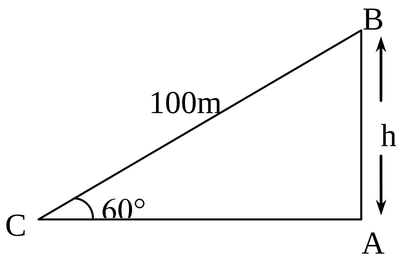
$$\frac{AB}{AC} = \sin 60°$$

$$\frac{60}{AC} = \frac{\sqrt{3}}{2} \quad \text{or} \quad AC = \frac{120}{\sqrt{3}} = 40\sqrt{3}\text{m}.$$

Hence the length of the string is $40\sqrt{3}$ metres.

Example 35. The string of a kite is 100 metres long and it make and angle of 60° with the horizontal. Find the height of the kite assuming that there is no slack in the string.

Sol : Let CA be the horizontal ground and let B be the position of the kite at a height h above the ground. The AB = h.



In right angle ΔCAB

$$\frac{AB}{CB} = \sin 60°$$

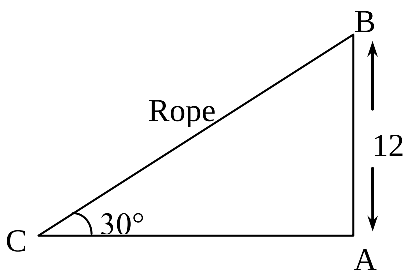
$$\frac{h}{100} = \frac{\sqrt{3}}{2} \quad \text{or} \quad h = 100\frac{\sqrt{3}}{2}$$

$\therefore$ h = $50\sqrt{3}$ metres

Hence the height of the kite is $50\sqrt{3}$ metres.

Example 36. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at a ground. The height of the pole is 12m and the angle made by the rope with the ground level is 30°. Calculate the distance covered by the artist in climbing to the top of the pole ?

Sol : Let vertical pole AB of height in metres and CB be the rope.



In right angle ΔCAB

$$\frac{AB}{CB} = \sin\theta = \sin 30°$$

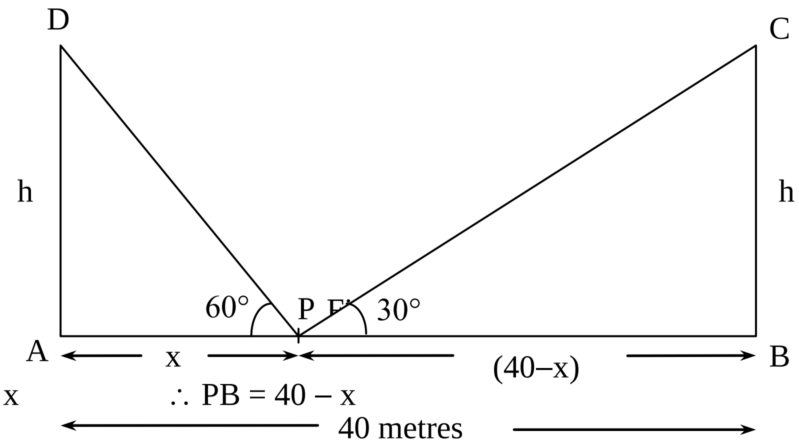
$$\frac{12}{CB} = \frac{1}{2} \qquad \therefore \ CB = 24 \text{ m}$$

Hence the distance covered by the circus artist is 24 m.

Example 37. Two polls of equal height stand on either side of a roadways which is 40 metres wide at a point in the roadway between the polls. The elevation of the tops of the polls are 60° and 30°. Find their height and the position of the point ?

Sol : let AB = 40 metres be the width at the roadway. Let AD = h, BC = h metres be the two polls. Let P be any point on AB at which the ngle of elevation of the tops are 60° and 30°.

Then $\angle APD = 60°$ and $\angle BPC = 30°$



Let AP = x $\quad \therefore$ PB = 40 − x

Now from right angle Δ PBC

$$\frac{BC}{BP} = \tan 30° \qquad \text{or} \qquad \frac{h}{40 - x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}h = 40 - x \qquad \ldots\ldots (i)$$

Again from right angle ΔPAD

$$\frac{AD}{PA} = \tan 60° \qquad \text{or} \qquad \frac{h}{x} = \sqrt{3}$$

$$\therefore \qquad h = \sqrt{3}x \qquad \ldots\ldots (ii)$$

Substituting the value of h in eqn**.** (i), we get

$$\sqrt{3} \times \sqrt{3}x = 40 - x \qquad \text{or} \qquad 3x = 40 - x$$

$4x = 40$ so $x = 10$ metres.

If x = 10 metres than h = $\sqrt{3}.10$ = 17.32 metres (height of polls)

Hence the point P divides AB in the ratio 1 : 3.

EXERCISE- VI

1. A tower stands vertically on the ground from a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 60°. What is the height of the tower?
2. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5m away from the wall. Find the length of the ladder.
3. A ladder is placed along a wall of a house such that upper end is touching the top of the wall. The foot of the ladder is 2m away from the wall and ladder is making an angle of 60° with the level of the ground. Find the height of the wall?
4. A telephone pole is 10 m high. A steel wire tied to tope of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle 45° with the horizontal through the foot of the pole, find the length of the wire.
5. A kite is flying at a height of 75 metres from the ground level, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest metre.
6. A vertical tower stands on a horizontal plane a is surmounted by a vertical flag–staff. At a point on the plane 70 metres away from the tower, an observer notices that the angle of elevation of the top and bottom of the flag–staff are respectively 60° and 45°. Find the height of the flag–staff and that of the tower. A circus artist is climbing a 20 metre long rope which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.
7. A person standing on the bank of a river, observer that the angle subtended by a tree on the opposite bank is 60°. When the retreats 20 metres from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river?
8. The angle of elevation of the top of the building at a distance of 50 m from its foot on a horizontal plane is found to be 60°. Find the height of the building.
9. From the top of the tower 30 m height a man is observing the base of a tree at an angle of depression measuring 30°. Find the distance between the tree and the tower.

10. The length of a string between a kite and a point on the ground is 90 m. If the string is making an angle θ with the level ground such that tan θ = 15/8, how high will the kite be?

ANSWERS

1. $20\sqrt{3}$	2. 19 m	3. $2\sqrt{3}$m
4. 14.1 m	5. 87 m	6. 51.24m and 70m
7. 10 mtrs, 17.32 mtrs.	8. 86.6 m.	9. 51.96 m
10. 79.41m		

UNIT IV
CO-ORDINATE GEOMETRY

Learning Objectives

- To understand the basic concepts of two dimensional coordinate geometry with points and straight lines.
- To learn different forms of straight lines with different methods to understand them.

4.1 POINTS

Cartesian Plane: Let XOX' and YOY' be two perpendicular lines. 'O' be their intersecting point called origin. XOX' is horizontal line called X-axis and YOY' is vertical line called Y-axis. The plane made by these axes is called Cartesian plane or coordinate plane.

The axes divide the plane into four parts called quadrant: 1st quadrant, 2nd quadrant, 3rd quadrant and 4th quadrant as shown in the Fig. 4.1. OX is known as positive direction of X-axis and OX' is known as negative direction of X-axis. Similarly, OY is known as positive direction of Y-axis and OY' is known as negative direction of Y-axis.

The axes XOX' and YOY' are together known as rectangular axes or coordinate axes.

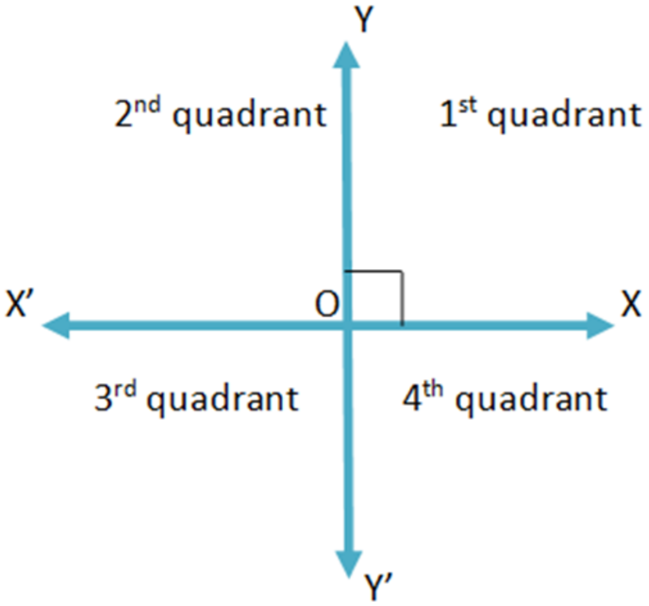


Fig. 4.1

Point: A point is a mark of location on a plane. It has no dimension i.e. no length, no breadth and no height. For example, tip of pencil, toothpick etc. A point in a plane is represented as an ordered pair of real numbers called coordinates of point.

The perpendicular distance of a point from the Y-axis is $P(x, y)$ called abscissa or x-coordinate and the perpendicular distance of a point from the X-axis is called ordinate or y-coordinate. If $P(x, y)$ be any point in the plane (see Fig. 4.2) then x is the abscissa of the point P and y is the ordinate of the point P.

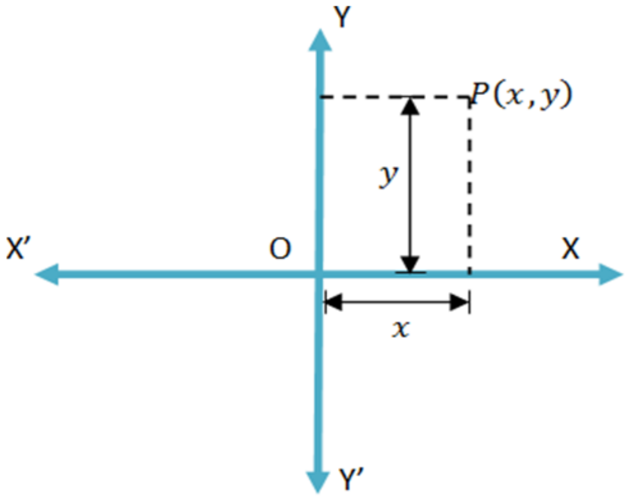


Fig. 4.2

Note: **(i)** If distance along X-axis is measured to the right of Y-axis then it is positive and if it is measured to the left of Y-axis then it is negative.

(ii) If distance along Y-axis is measured to the above of X-axis then it is positive and if it is measured to the below of X-axis then it is negative.

(iii) The coordinates of origin 'O' are $(0,0)$.

(iv) A point on X-axis is represented as $(x, 0)$ i.e. ordinate is zero.

(v) A point on Y-axis is represented as $(0, y)$ i.e. abscissa is zero.

(vi) In the 1st quadrant $x > 0$ and $y > 0$

In the 2nd quadrant $x < 0$ and $y > 0$

In the 3rd quadrant $x < 0$ and $y < 0$

In the 4th quadrant $x > 0$ and $y < 0$.

Distance between Two Points in a Plane: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points in a plane then the distance between these points is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1. Plot the following points and find the quadrant in which they lie:

(i) $A(2,2)$ (ii) $B(-3,-1)$ (iii) $C(-1,3)$ (iv) $D(3,-2)$

Sol.

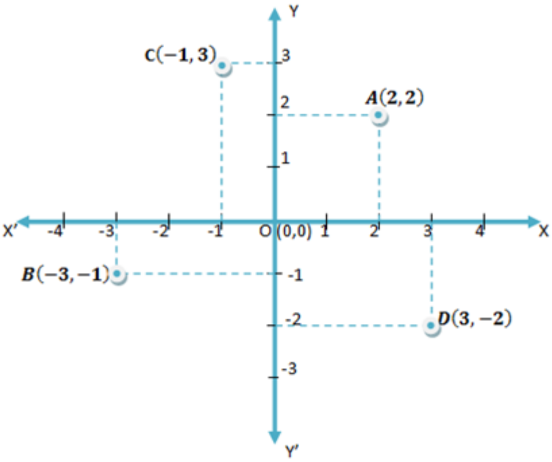


Fig. 4.3

In the Fig. 4.3, it is clear that

(i) Point $\boldsymbol{A(2,2)}$ lies in the 1st quadrant.
(ii) Point $\boldsymbol{B(-3,-1)}$ lies in the 3rd quadrant.
(iii) Point $\boldsymbol{C(-1,3)}$ lies in the 2nd quadrant.
(iv) Point $\boldsymbol{D(3,-2)}$ lies in the 4th quadrant.

Example 2. Without plotting, find the quadrant in which the following points lie:

(i) $A(2,-3)$ (ii) $B(-5,-6)$ (iii) $\mathrm{C}(4,3)$ (iv) $D(-1,5)$
(v) $E(0,9)$ (vi) $F(-3,0)$ (vii) $\mathrm{G}(0,-7)$ (viii) $H(1,0)$

Sol.

(i) The given point is $A(2,-3)$

Here X-coordinate $= 2$, which is positive and Y-coordinate $= -3$, which is negative.

Hence the point $A(2,-3)$ lies in 4th quadrant.

(ii) The given point is $B(-5,-6)$

Here X-coordinate $= -5$, which is negative and Y-coordinate $= -6$, which is also negative.

Hence the point $B(-5,-6)$ lies in 3rd quadrant.

(iii) The given point is $\mathrm{C}(4,3)$

Here X-coordinate = 4 > 0 and Y-coordinate = 3 > 0.

Hence the point $\mathrm{C}(4,3)$ lies in 1st quadrant.

(iv) The given point is $D(-1,5)$

Here X-coordinate $= -1 < 0$ and Y-coordinate = 5 > 0.

Hence the point $D(-1,5)$ lies in 2nd quadrant.

(v) The given point is $E(0,9)$

Here X-coordinate = 0 and Y-coordinate = 9 > 0.

Hence the point $E(0,9)$ lies on Y-axis above the origin.

(vi) The given point is $F(-3,0)$

Here X-coordinate $= -3 < 0$ and Y-coordinate = 0.

Hence the point $F(-3,0)$ lies on X-axis left to origin.

(vii) The given point is $G(0,-7)$

Here X-coordinate = 0 and Y-coordinate $= -7 < 0$.

Hence the point $G(0,-7)$ lies on Y-axis below the origin.

(viii) The given point is $H(1,0)$

Here X-coordinate = 1 > 0 and Y-coordinate = 0.

Hence the point $H(1,0)$ lies on X-axis right to origin.

Example 3. Find the distance between the following pairs of points:

(i) $(0,5)\,,(3,6)$ (ii) $(-1,2)\,,(4,3)$ (iii) $(2,0)\,,(-3,-2)$

(iv) $(1,2)\,,(4,5)$ (v) $(-2,3)\,,(-5,7)$ (vi) $(-1,-3)\,,(-2,-4)$

(vii) $(a-b, c-d)\,,(-b+c, c+d)$

(viii) $(sin\theta, cos\theta)\,,(-sin\theta, cos\theta)$

Sol.

(i) Let **A** represents the point $(0,5)$ and **B** represents the point $(3,6)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(3-0)^2+(6-5)^2}$$
$$=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}\,units$$

(ii) Let **A** represents the point $(-1,2)$ and **B** represents the point $(4,3)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(4-(-1))^2+(3-2)^2}$$
$$=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}\,units$$

(iii) Let **A** represents the point $(2,0)$ and **B** represents the point $(-3,-2)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-3-2)^2+(-2-0)^2}$$
$$=\sqrt{(-5)^2+(-2)^2}=\sqrt{25+4}=\sqrt{29}\,units$$

(iv) Let **A** represents the point $(1,2)$ and **B** represents the point $(4,5)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(4-1)^2+(5-2)^2}$$
$$=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}$$
$$=\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}\,units$$

(v) Let **A** represents the point $(-2,3)$ and **B** represents the point $(-5,7)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-5-(-2))^2+(7-3)^2}$$
$$=\sqrt{(-5+2)^2+(4)^2}=\sqrt{(-3)^2+(4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5\,units$$

(vi) Let **A** represents the point $(-1,-3)$ and **B** represents the point $(-2,-4)$.

So, the distance between **A** and **B** is:

$$AB = \sqrt{(-2-(-1))^2 + (-4-(-3))^2}$$
$$= \sqrt{(-2+1)^2 + (-4+3)^2} = \sqrt{(-1)^2 + (-1)^2}$$
$$= \sqrt{1+1} = \sqrt{2}\,units$$

(vii) Let **A** represents the point $(a-b, c-d)$ and **B** represents the point $(-b+c, c+d)$.

So, the distance between **A** and **B** is:

$$AB = \sqrt{(-b+c-(a-b))^2 + (c+d-(c-d))^2}$$
$$= \sqrt{(-b+c-a+b)^2 + (c+d-c+d)^2}$$
$$= \sqrt{(c-a)^2 + (d+d)^2} = \sqrt{c^2 + a^2 - 2ac + (2d)^2}$$
$$= \sqrt{c^2 + a^2 + 4d^2 - 2ac}\ \ units$$

(viii) Let **A** represents the point $(sin\theta, cos\theta)$ and **B** represents the point $(-sin\theta, cos\theta)$.

So, the distance between **A** and **B** is:

$$AB = \sqrt{(-\sin\theta - \sin\theta)^2 + (\cos\theta - \cos\theta)^2}$$
$$= \sqrt{(-2\sin\theta)^2 + (0)^2} = \sqrt{4\sin^2\theta}$$
$$= 2\sin\theta\,units$$

Example 4. Using distance formula, prove that the triangle formed by the points $A(4,0)$, $B(-1,-1)$ and $C(3,5)$ is an isosceles triangle.

Sol. Given that vertices of the triangle are $A(4,0)$, $B(-1,-1)$ and $C(3,5)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB = \sqrt{(4-(-1))^2 + (0-(-1))^2}$$
$$= \sqrt{(5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26}\,units$$

Distance between **B** and **C** is

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{16+36} = \sqrt{52}\,units$$

Distance between **A** and **C** is

$$AC = \sqrt{(4-3)^2 + (0-5)^2}$$
$$= \sqrt{(1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}\,units$$

We can see that $AB = AC \neq BC$

Hence the triangle formed by the points $A(4,0)$, $B(-1,-1)$ and $C(3,5)$ is an isosceles triangle.

Example 5. Using distance formula, prove that the triangle formed by the points $A(0,0)$, $B(0,2)$ and $C(\sqrt{3}, 1)$ is an equilateral triangle.

Sol. Given that vertices of the triangle are $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB=\sqrt{(0-0)^2+(0-2)^2}$$
$$=\sqrt{(0)^2+(-2)^2}=\sqrt{0+4}=\sqrt{4}=2\,units$$

Distance between **B** and **C** is

$$BC=\sqrt{\left(0-\sqrt{3}\right)^2+(2-1)^2}$$
$$=\sqrt{\left(-\sqrt{3}\right)^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2\,units$$

Distance between **A** and **C** is

$$AC=\sqrt{\left(0-\sqrt{3}\right)^2+(0-1)^2}$$
$$=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}=\sqrt{3+1}=\sqrt{4}=2\,units$$

We can see that $AB=BC=AC$

Hence the triangle formed by the points $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$ is an equilateral triangle.

<u>Mid-point between two points</u>: If $A(x_1,y_1)$ and $B(x_2,y_2)$ be any two points then the mid-point between these points is given by:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Example 6. Find the mid points between the following pairs of points:

(i) $(2,3)$, $(8,5)$ (ii) $(6,3)$, $(6,-9)$ (iii) $(-2,-4)$, $(3,-6)$

(iv) $(0,8)$, $(6,0)$ (v) $(0,0)$, $(-12,10)$ (vi) (a,b) , (c,d)

(vii) $(a+b,c-d)$, $(-b+3a,c+d)$

Sol.

(i) The given points are $(2,3)$ and $(8,5)$.

So, the mid-point between these points is given by:

$$\left(\frac{2+8}{2},\frac{3+5}{2}\right)=\left(\frac{10}{2},\frac{8}{2}\right)=(5,4)$$

(ii) The given points are $(6,3)$ and $(6,-9)$.

So, the mid-point between these points is given by:

$$\left(\frac{6+6}{2},\frac{3+(-9)}{2}\right)=\left(\frac{12}{2},\frac{3-9}{2}\right)=\left(\frac{12}{2},\frac{-6}{2}\right)=(6,-3)$$

(iii) The given points are $(-2,-4)$ and $(3,-6)$.

So, the mid-point between these points is given by:

$$\left(\frac{-2+3}{2},\frac{-4+(-6)}{2}\right)=\left(\frac{1}{2},\frac{-4-6}{2}\right)=\left(\frac{1}{2},\frac{-10}{2}\right)=\left(\frac{1}{2},-5\right)$$

(iv) The given points are $(0,8)$ and $(6,0)$.

So, the mid-point between these points is given by:

$$\left(\frac{0+6}{2},\frac{8+0}{2}\right)=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$$

(v) The given points are $(0,0)$ and $(-12,10)$.

So, the mid-point between these points is given by:

$$\left(\frac{0+(-12)}{2},\frac{0+10}{2}\right)=\left(\frac{-12}{2},\frac{10}{2}\right)=(-6,5)$$

(vi) The given points are (a,b) and (c,d).

So, the mid-point between these points is given by:

$$\left(\frac{a+c}{2},\frac{b+d}{2}\right)$$

(vii) The given points are $(a+b,c-d)$ and $(-b+3a,c+d)$.

So, the mid-point between these points is given by:

$$\left(\frac{a+b-b+3a}{2},\frac{c-d+c+d}{2}\right)=\left(\frac{4a}{2},\frac{2c}{2}\right)=(2a,c)$$

Example 7. If the mid-point between two points is $(3,5)$ and one point between them is $(-1,2)$, find the other point.

Sol. Let the required point is (a,b).

So, according to given statement $(3,5)$ is the mid-point of $(-1,2)$ and (a,b).

$$\Rightarrow \quad (3,5)=\left(\frac{-1+a}{2},\frac{2+b}{2}\right)$$

$$\Rightarrow \quad \frac{-1+a}{2}=3 \quad \& \quad \frac{2+b}{2}=5$$

$$\Rightarrow \quad -1+a=6 \quad \& \quad 2+b=10$$

$$\Rightarrow \quad a=7 \quad \& \quad b=8$$

Hence the required point is $(7,8)$.

Example 8. If the mid-point between two points is $(-7,6)$ and one point between them is $(3,-9)$, find the other point.

Sol. Let the required point is (a,b).

So, according to given statement $(-7,6)$ is the mid-point of $(3,-9)$ and (a,b).

$$\Rightarrow \quad (-7,6)=\left(\frac{3+a}{2},\frac{-9+b}{2}\right)$$

$$\Rightarrow \quad \frac{3+a}{2}=-7 \quad \& \quad \frac{-9+b}{2}=6$$

$$\Rightarrow \quad 3+a=-14 \quad \& \quad -9+b=12$$

$$\Rightarrow \quad a=-17 \quad \& \quad b=21$$

Hence the required point is $(-17,21)$.

Centroid of a Triangle: The centroid of a triangle is the intersection point of the three medians of the triangle. In other words, the **average** of the three vertices of the triangle is called the centroid of the triangle.

i.e. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are three vertices of a triangle then the centroid of the triangle is given by:

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

In the Fig. 4.4, the point G is the centroid of the Triangle.

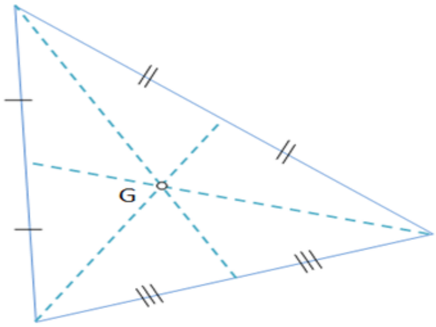


Fig. 4.4

Example 9. Vertices of the triangles are given below, find the centroid of the triangles:

(i) $(5,2)$, $(5,4)$, $(8,6)$

(ii) $(4,-3)$, $(-4,8)$, $(5,7)$

(iii) $(2,-4)$, $(0,-10)$, $(4,5)$

(iv) $(9,-9)$, $(5,8)$, $(-7,-2)$

Sol.

(i) The given vertices of the triangle are $(5,2)$, $(5,4)$ and $(8,6)$.

So, the centroid of the triangle is

$$\left(\frac{5+5+8}{3}, \frac{2+4+6}{3}\right) = \left(\frac{18}{3}, \frac{12}{3}\right) = (6,4)$$

(ii) The given vertices of the triangle are $(4,-3)$, $(-4,8)$ and $(5,7)$.

So, the centroid of the triangle is

$$\left(\frac{4-4+5}{3}, \frac{-3+8+7}{3}\right) = \left(\frac{5}{3}, \frac{12}{3}\right) = \left(\frac{5}{3}, 4\right)$$

(iii) The given vertices of the triangle are $(2,-4)$, $(0,-10)$ and $(4,5)$.

So, the centroid of the triangle is

$$\left(\frac{2+0+4}{3}, \frac{-4-10+5}{3}\right) = \left(\frac{6}{3}, \frac{-9}{3}\right) = (2,-3)$$

(iv) The given vertices of the triangle are $(9,-9)$, $(5,8)$ and $(-7,-2)$.

So, the centroid of the triangle is

$$\left(\frac{9+5-7}{3}, \frac{-9+8-2}{3}\right) = \left(\frac{7}{3}, \frac{-3}{3}\right) = \left(\frac{7}{3}, -1\right)$$

Example 10. If centroid of the triangle is $(10,18)$ and two vertices of the triangle are $(1,-5)$ and $(3,7)$, find the third vertex of the triangle.

Sol. Let the required vertex of the triangle is (a, b).

So, according to given statement and definition of centroid, we get

$$\Rightarrow \quad (10,18)=\left(\frac{1+3+a}{3},\frac{-5+7+b}{3}\right)$$

$$\Rightarrow \quad \frac{1+3+a}{3}=10 \quad \& \quad \frac{-5+7+b}{3}=18$$

$$\Rightarrow \quad 1+3+a=30 \quad \& \quad -5+7+b=54$$

$$\Rightarrow \quad a=26 \quad \& \quad b=52$$

Hence the required vertex of triangle is $(26,52)$.

Example 11. If centroid of the triangle is $(-5,-7)$ and two vertices of the triangle are $(0,6)$ and $(-3,2)$, find the third vertex of the triangle.

Sol. Let the required vertex of the triangle is (a,b).

So, according to given statement and definition of centroid, we get

$$\Rightarrow \quad (-5,-7)=\left(\frac{0-3+a}{3},\frac{6+2+b}{3}\right)$$

$$\Rightarrow \quad \frac{0-3+a}{3}=-5 \quad \& \quad \frac{6+2+b}{3}=-7$$

$$\Rightarrow \quad 0-3+a=-15 \quad \& \quad 6+2+b=-21$$

$$\Rightarrow \quad a=-12 \quad \& \quad b=-29$$

Hence the required vertex of triangle is $(-12,-29)$.

Example 12. If centroid of a triangle formed by the points $(1,a)$, $(9,b)$ and $(c^2,-5)$ lies on the X-axis, prove that $a+b=5$.

Sol. Given that vertices of the triangle are $(1,a)$, $(9,b)$ and $(c^2,-5)$.

Centroid of the triangle is given by

$$\left(\frac{1+9+c^2}{3},\frac{a+b-5}{3}\right)$$

By given statement, the centroid of the triangle lies on the X-axis.

Therefore, Y-coordinate of centroid is zero.

$$\Rightarrow \quad \frac{a+b-5}{3}=0$$

$$\Rightarrow \quad a+b-5=0$$

$$\Rightarrow \quad a+b=5$$

Example 13. If centroid of a triangle formed by the points $(-a,a)$, (c^2,b) and $(d,5)$ lies on the Y-axis, prove that $c^2=a-d$.

Sol. Given that vertices of the triangle are $(-a,a)$, (c^2,b) and $(d,5)$.

Centroid of the triangle is given by

$$\left(\frac{-a+c^2+d}{3},\frac{a+b+5}{3}\right)$$

By given statement, the centroid of the triangle lies on the Y-axis.

Therefore, X-coordinate of centroid is zero.

$\Rightarrow \qquad \dfrac{-a+c^2+d}{3}=0$

$\Rightarrow \qquad -a+c^2+d=0$

$\Rightarrow \qquad c^2=a-d$

Hence proved.

EXERCISE-I

1. The point $(-3,-4)$ lies in the quadrant:

   (a) First (b) Second (c) Third (d) Fourth

2. The point $(7,4)$ lies in the quadrant

   (a) First (b) Second (c) Third (d) Fourth

3. Find the distance between the following pairs of points:

   (i) $(-1,2)$, $(4,3)$ (ii) $(a-b,c-d)$, $(-b+c,c+d)$

4. Find the mid points between the following pairs of points:

   (i) $(0,8)$, $(6,0)$ (ii) $(a+b,c-d)$, $(-b+3a,c+d)$

5. The mid-point between two points is $(3,5)$ and one point between them is $(-1,2)$. Find the other point.

6. Find the centroids of triangles whose vertices are:

   (i) $(4,-3)$, $(-4,8)$, $(5,7)$ (ii) $(9,-9)$, $(5,8)$, $(-7,-2)$

<u>ANSWERS</u>

1. (c)
2. (a)
3. (i) $\sqrt{26}$, (ii) $\sqrt{c^2+a^2+4d^2-2ac}$
4. (i) $(3,4)$, (ii) $(2a,c)$
5. $(7,8)$
6. (i) $\left(\frac{5}{3},4\right)$, (ii) $\left(\frac{7}{3},-1\right)$

4.2 STRAIGHT LINES

<u>Definition of Straight Line</u>: A path traced by a moving point travelling in a constant direction is called a straight line.

OR

The shortest distance between two points in a plane is called a straight line.

<u>General Equation of Straight Line</u>: A straight line in XY plane has general form

$$ax+by+c=0$$

where a is the coefficient of x, b is the coefficient of y and c is the constant term.

Note: **(i)** Any point (x_1, y_1) lies on the line $ax + by + c = 0$ if it satisfies the equations of the line **i.e.** if we substitute the values x_1 at the place of x and y_1 at the place of y in the equation of line, the result $ax_1 + by_1 + c$ becomes zero.

(ii) X-axis is usually represented horizontally and its equation is $y = 0$.

(iii) Y-axis is usually represented vertically and its equation is $x = 0$.

(vi) $x = k$ represents the line parallel to Y-axis, where k is some constant .

(v) $y = k$ represents the line parallel to X-axis, where k is some constant .

Slope of a Straight Line: Slope of straight line measures how slanted the line is relative to the horizontal (see Fig. 4.5). It is usually represented by $\boldsymbol{m}$.

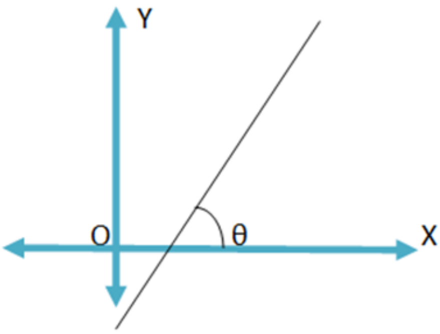


Fig. 4.5

To find Slope of a Straight Line:

(i) If a line making an angle θ with positive X-axis then the slope m of the line is given by $m = tan\theta$.

(ii) If a line passes through two points (x_1, y_1) and (x_2, y_2) then the slope m of the line is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(iii) If equation of a straight line is $ax + by + c = 0$, then its slope m is given by $m = -\frac{a}{b}$.

Note: **(i)** Slope of a horizontal line is always zero **i.e.** slope of a line parallel to X-axis is zero as $m = tan0° = 0$.

(ii) Slope of a vertical line is always infinity **i.e.** slope of a line perpendicular to X-axis is infinity as $= tan90° = \infty$.

(iii) Let L_1 and L_2 represents two straight lines. Let m_1 and m_2 be slopes of L_1 and L_2 respectively. We say that L_1 and L_2 are parallel lines iff $m_1 = m_2$ **i.e.** slopes are equal. We say that L_1 and L_2 are perpendicular iff $m_1 . m_2 = -1$ **i.e.** product of slopes is equal to -1 (*except the cases of axes and lines parallel to axes*).

Example 14. Find the slope of the straight lines which make following angles:

(i) 45° (ii) 120° (iii) 30° (iv) 150° (v) 210°

with the positive direction of X-axis.

Sol.

(i) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 45°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 45^\circ$

$\Rightarrow \quad m = 1$

which is the required slope.

(ii) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 120°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 120^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 60^\circ\right)$

$\Rightarrow \quad m = -\tan\left(60^\circ\right)$

$\Rightarrow \quad m = -\sqrt{3}$

which is the required slope.

(iii) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 30°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 30^\circ$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$

which is the required slope.

(iv) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 150°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 150^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 30^\circ\right)$

$\Rightarrow \quad m = -\tan\left(30^\circ\right)$

$\Rightarrow \quad m = -\frac{1}{\sqrt{3}}$

which is the required slope.

(v) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 210°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 210^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ + 30^\circ\right)$

$\Rightarrow \quad m = \tan\left(30^\circ\right)$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$

which is the required slope.

Example 15. Find the slope of the straight lines which pass through the following pairs of points:

(i) $(2,5)$, $(6,17)$ (ii) $(-8,7)$, $(3,-5)$

(iii) $(0,-6)$, $(7,9)$ (iv) $(-11,-5)$, $(-3,-10)$

(v) $(0,0)$, $(10,-12)$.

Sol.

(i) Given that the straight line passes through the points $(2,5)$ and $(6,17)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 2$, $y_1 = 5$, $x_2 = 6$ and $y_2 = 17$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{17-5}{6-2} = \frac{12}{4}$

$\Rightarrow \quad m = 3$

which is the required slope.

(ii) Given that the straight line passes through the points $(-8,7)$ and $(3,-5)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = -8$, $y_1 = 7$, $x_2 = 3$ and $y_2 = -5$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{-5-7}{3-(-8)} = \frac{-12}{3+8}$

$\Rightarrow \quad m = -\frac{12}{11}$

which is the required slope.

(iii) Given that the straight line passes through the points $(0,-6)$ and $(7,9)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0$, $y_1 = -6$, $x_2 = 7$ and $y_2 = 9$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{9-(-6)}{7-0} = \frac{9+6}{7}$

$\Rightarrow \qquad m = \frac{15}{7}$

which is the required slope.

(iv) Given that the straight line passes through the points $(-11, -5)$ and $(-3, -10)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = -11$, $y_1 = -5$, $x_2 = -3$ and $y_2 = -10$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \qquad m = \frac{-10-(-5)}{-3-(-11)} = \frac{-10+5}{-3+11}$

$\Rightarrow \qquad m = -\frac{5}{8}$

which is the required slope.

(v) Given that the straight line passes through the points $(0,0)$ and $(10, -12)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0$, $y_1 = 0$, $x_2 = 10$ and $y_2 = -12$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \qquad m = \frac{-12-0}{10-0} = \frac{-12}{10}$

$\Rightarrow \qquad m = -\frac{6}{5}$

which is the required slope.

Example 16. Find the slopes of the following straight lines:

(i) $2x + 4y + 5 = 0$ **(ii)** $x - 3y + 9 = 0$

(iii) $5y - 10x + 1 = 0$ **(iv)** $-2x - 6y = 0$

(v) $x = 5$ **(vi)** $y = -6$

Sol.

(i) Given that equation of the straight line is $2x + 4y + 5 = 0$.

Comparing this equation with $ax + by + c = 0$, we get

$a = 2$, $b = 4$ and $c = 5$

Let m be the slope of given straight line.

Therefore, $m = -\frac{a}{b}$

$\Rightarrow \qquad m = -\frac{2}{4}$

$\Rightarrow \qquad m = -\frac{1}{2}$

which is the required slope.

(ii) Given that equation of the straight line is $-3y + 9 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = 1$, $b = -3$ and $c = 9$
Let m be the slope of given straight line.
Therefore, $m=-\left(\frac{1}{-3}\right)$

$\Rightarrow \quad m=\frac{1}{3}$

which is the required slope.

(iii) Given that equation of the straight line is $5y - 10x + 1 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = -10$, $b = 5$ and $c = 1$
Let m be the slope of given straight line.
Therefore, $m=-\left(\frac{-10}{5}\right)$

$\Rightarrow \quad m=2$

which is the required slope.

(iv) Given that equation of the straight line is $-2x - 6y = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = -2$, $b = -6$ and $c = 0$
Let m be the slope of given straight line.
Therefore, $m=-\frac{a}{b}$

$\Rightarrow \quad m=-\left(\frac{-2}{-6}\right)$

$\Rightarrow \quad m=-\frac{1}{3}$

which is the required slope.

(v) Given that equation of the straight line is $x = 5$.
This equation is parallel to Y-axis.
Hence the slope of the line is infinity.

(vi) Given that equation of the straight line is $y = -6$.
This equation is parallel to X-axis.
Hence the slope of the line is zero.

Example 17. Find the equation of straight line which is parallel to X-axis passes through $(1,5)$.

Sol. Equation of straight line parallel to X-axis is given by

$$y=k \qquad (1)$$

Given that the straight line passes through the point $(1,5)$.

Put $x = 1$ and $y = 5$ in equation (1), we get

$$5=k$$

So, $y=5$ be the required equation of straight line.

Example 18. Find the equation of straight line which is parallel to Y-axis passes through $(-3, -7)$.

Sol. Equation of straight line parallel to Y-axis is given by

$$x = k \quad (1)$$

Given that the straight line passes through the point $(-3, -7)$.

Put $x = -3$ and $y = -7$ in equation (1), we get

$$-3 = k$$

So, $x = -3$ be the required equation of straight line.

Equation of Straight Line Passing Through Origin:

If a line passes through origin and m be its slope. $P(x, y)$ be any point on the line (see Fig. 4.6), then equation of straight line is $\boldsymbol{y = mx}$.

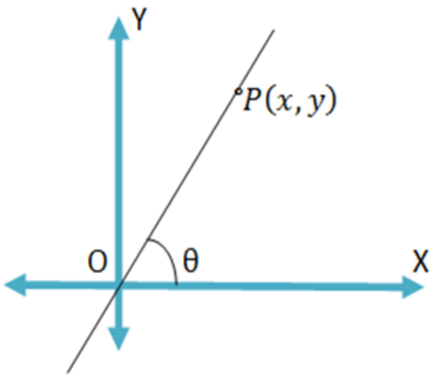


Fig. 4.6

(*except the cases of Y-axis and parallel to Y-axis*)

Example 19. Find the equation of straight line having slope equal to 5 and passes through origin.

Sol. Let m be the slope of required line. Therefore $m = 5$.

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = 5x$ be the required equation of straight line.

Example 20. Find the equation of straight line having slope equal to -10 and passes through origin.

Sol. Let m be the slope of required line. Therefore $m = -10$.

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = -10x$ be the required equation of straight line.

Example 21. Find the equation of straight line which passes through origin and makes an angle $60°$ with the positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 60°$

$\Rightarrow \quad m = \sqrt{3}$

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = \sqrt{3}x$ be the required equation of straight line.

Example 22. Find the equation of straight line which passes through origin and makes an angle $135°$ with the positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 135°$

$\Rightarrow \quad m = \tan(180° - 45°)$

$\Rightarrow \quad m = -\tan 45°$

$\Rightarrow \quad m = -1$

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = -x$ be the required equation of straight line.

<u>Equation of Straight Line in Point-Slope form</u>:

If a line passes through a point (x_1, y_1), m be its slope and $P(x, y)$ be any point on the line (see Fig. 4.7), then equation of straight line is $\boldsymbol{y - y_1 = m(x - x_1)}$.

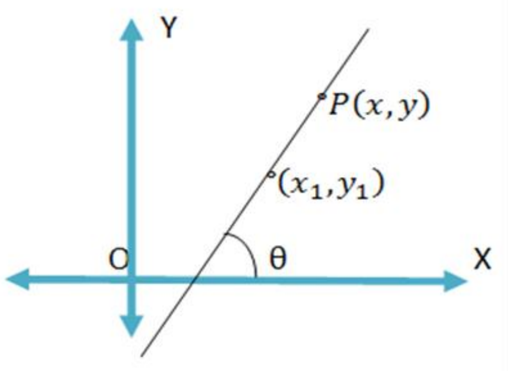


Fig. 4.7

(*except the cases of Y-axis and parallel to Y-axis*)

Example 23. Find the equation of straight line having slope equal to 9 and passes through the point (1,5).

Sol. Let m be the slope of required line. Therefore $m = 9$.

Also it is given that the required line passes through the point $(1,5)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - 5 = 9(x - 1)$
$\Rightarrow \quad y - 5 = 9x - 9$
$\Rightarrow \quad 9x - y - 9 + 5 = 0$
$\Rightarrow \quad 9x - y - 4 = 0$

which is the required equation of straight line.

Example 24. Find the equation of straight line passes through $(-4, -2)$ and having slope -8.

Sol. Let m be the slope of required line. Therefore $m = -8$.

Also it is given that the required line passes through the point $(-4, -2)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - (-2) = -8(x - (-4))$
$\Rightarrow \quad y + 2 = -8(x + 4)$
$\Rightarrow \quad y + 2 = -8x - 32$
$\Rightarrow \quad 8x + y + 2 + 32 = 0$
$\Rightarrow \quad 8x + y + 34 = 0$

which is the required equation of straight line.

Example 25. Find the equation of straight line passes through $(0, -8)$ and makes an angle $30°$ with positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 30^\circ$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$

Also it is given that the required line passes through the point $(0, -8)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - (-8) = \frac{1}{\sqrt{3}}(x - 0)$

$\Rightarrow \quad y + 8 = \frac{x}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}y + 8\sqrt{3} = x$

$\Rightarrow \quad x - \sqrt{3}y - 8\sqrt{3} = 0$

which is the required equation of straight line.

Example 26. Find the equation of straight line passes through $(-9,0)$ and makes an angle $150°$ with positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 150^\circ$

$\Rightarrow \quad m = \tan(180^\circ - 30^\circ)$

$$\Rightarrow \qquad m = -\tan(30^\circ)$$

$$\Rightarrow \qquad m = -\frac{1}{\sqrt{3}}$$

Also it is given that the required line passes through the point $(-9,0)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$$\Rightarrow \qquad y - 0 = -\frac{1}{\sqrt{3}}(x - (-9))$$

$$\Rightarrow \qquad -\sqrt{3}\,y = x + 9$$

$$\Rightarrow \qquad x + \sqrt{3}\,y + 9 = 0$$

which is the required equation of straight line.

Equation of Straight Line in Two Points form:

If a line passes through two points (x_1, y_1) and (x_2, y_2) and $P(x, y)$ be any point on the line (see Fig. 4.8), then the equation of straight line is

$$\boldsymbol{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)} \qquad \text{where } \boldsymbol{x_1 \neq x_2}$$

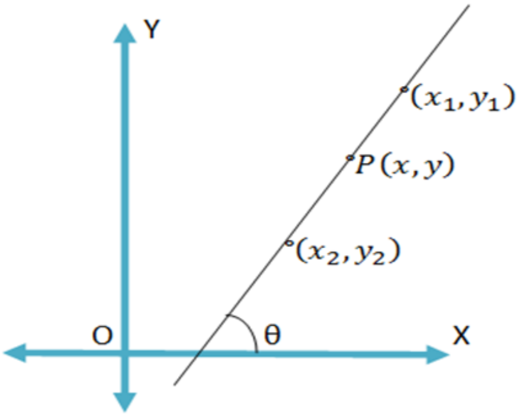


Fig. 4.8

Note: (i) In above case, if $x_1 = x_2$ then the equation of straight line is $x = x_1$.

Example 27. Find the equation of straight line passes through the points $(2, -2)$ and $(0,6)$.

Sol. Given that the straight line passes through the points $(2, -2)$ and $(0,6)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 2$, $y_1 = -2$, $x_2 = 0$ and $y_2 = 6$.

We know that equation of straight line in two points slope form is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

$$\Rightarrow \qquad y - (-2) = \left(\frac{6 - (-2)}{0 - 2}\right)(x - 2)$$

$\Rightarrow \quad y+2=\left(\frac{6+2}{-2}\right)(x-2)$

$\Rightarrow \quad y+2=-4(x-2)$

$\Rightarrow \quad y+2=-4x+8$

$\Rightarrow \quad 4x+y-6=0$

which is the required equation of straight line.

Example 28. Find the equation of straight line passes through the points $(0,8)$ and $(5,0)$.

Sol. Given that the straight line passes through the points $(0,8)$ and $(5,0)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0$, $y_1 = 8$, $x_2 = 5$ and $y_2 = 0$.

We know that equation of straight line in two points slope form is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1).$$

$\Rightarrow \quad y-8=\left(\frac{0-8}{5-0}\right)(x-0)$

$\Rightarrow \quad y-8=\frac{-8x}{5}$

$\Rightarrow \quad 5y-40=-8x$

$\Rightarrow \quad 8x+5y-40=0$

which is the required equation of straight line.

Example 29. Find the equation of straight line passes through the points $(7,-4)$ and $(-1,5)$.

Sol. Given that the straight line passes through the points $(7,-4)$ and $(-1,5)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 7$, $y_1 = -4$, $x_2 = -1$ and $y_2 = 5$.

We know that equation of straight line in two points slope form is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1).$$

$\Rightarrow \quad y-(-4)=\left(\frac{5-(-4)}{-1-7}\right)(x-7)$

$\Rightarrow \quad y+4=-\frac{9}{8}(x-7)$

$\Rightarrow \quad 8y+32=-9x+63$

$\Rightarrow \quad 9x+8y-31=0$

which is the required equation of straight line.

<u>Equation of Straight Line in Slope-Intercept form</u>:

If a line having slope m, its y-intercept is equal to c and $P(x, y)$ be any point on the line (see Fig. 4.9), then equation of straight line is $\boldsymbol{y = mx + c}$.

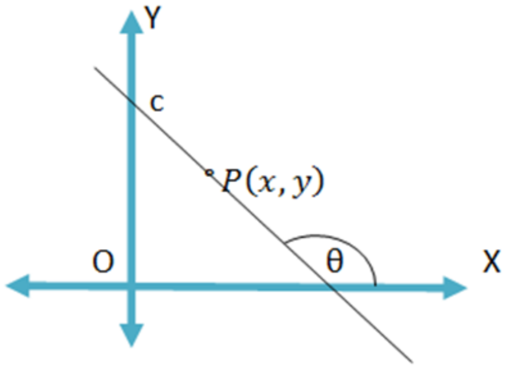


Fig. 4.9

<u>Note</u>: **(i)** If intercept c is given above the X-axis or above the origin then it is positive.

(ii) If intercept c is given below the X-axis or below the origin then it is negative.

Example 30. Find the equation of straight line having slope 3 and cuts of an intercept -2 on Y-axis.

Sol. Given that the slope m of straight line is 3 and Y-intercept is -2 i.e. $c = -2$.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = 3x - 2$$
$$\Rightarrow \quad 3x - y - 2 = 0$$

which is the required equation of straight line.

Example 31. Find the equation of straight line having slope -6 and cuts of an intercept 5 on Y-axis above the origin.

Sol. Given that the slope m of straight line is -6 and Y-intercept is 5 i.e. $c = 5$.

c is taken positive as Y-intercept is above the origin.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = -6x + 5$$
$$\Rightarrow \quad 6x + y - 5 = 0$$

which is the required equation of straight line.

Example 32. Find the equation of straight line having slope 2 and cuts of an intercept 9 on Y-axis below the origin.

Sol. Given that the slope m of straight line is 2 and Y-intercept is -9 i.e. $c = -9$.

c is taken negative as Y-intercept is below the origin.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = 2x - 9$$
$$\Rightarrow \quad 2x - y - 9 = 0$$

which is the required equation of straight line.

Example 33. Find the equation of straight line which makes an angle $45°$ with X-axis and cuts of an intercept 8 on Y-axis below the X-axis.

Sol. Given that the required line makes an angle $45°$ with X-axis.

Therefore slope m of straight line is given by $m=\tan 45^\circ$ *i.e.* $m=1$.

Also Y-intercept is -8 i.e. $c=-8$.

c is taken negative as Y-intercept is below the X-axis.

We know that equation of straight line in slope-intercept form is

$$y=mx+c$$
$$\Rightarrow \quad y=1x-8$$
$$\Rightarrow \quad x-y-8=0$$

which is the required equation of straight line.

Example 34. Find the equation of straight line which makes an angle $60°$ with X-axis and cuts of an intercept 5 on Y-axis above the X-axis.

Sol. Given that the required line makes an angle $60°$ with X-axis.

Therefore slope m of straight line is given by $m=\tan 60^\circ$ *i.e.* $m=\sqrt{3}$.

Also Y-intercept is 5 i.e. $c=5$.

c is taken positive as Y-intercept is above the X-axis.

We know that equation of straight line in slope-intercept form is

$$y=mx+c$$
$$\Rightarrow \quad y=\sqrt{3}x+5$$
$$\Rightarrow \quad \sqrt{3}x-y+5=0$$

which is the required equation of straight line.

Example 35. Find the equation of straight line which is parallel to the line passes through the points $(0,3)$ and $(2,0)$ and cuts of an intercept 12 on Y-axis below the origin.

Sol. Given that the required line is parallel to the line passes through the points $(0,3)$ and $(2,0)$.

Let m be the slope of the required line.

Therefore, $m=\frac{0-3}{2-0}$ *i.e.* $m=-\frac{3}{2}$ (as the slopes of parallel lines are same)

Also Y-intercept is -12 i.e. $c=-12$.

c is taken negative as Y-intercept is below the origin.

We know that equation of straight line in slope-intercept form is

$$y=mx+c$$
$$\Rightarrow \quad y=-\frac{3}{2}x-12$$
$$\Rightarrow \quad 2y=-3x-24$$
$$\Rightarrow \quad 3x+2y+24=0$$

which is the required equation of straight line.

Equation of Straight Line in Intercept form:

If a line having intercepts a and b on X-axis and Y-axis respectively and $P(x, y)$ be any point on the line (see figure 4.10), then equation of straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

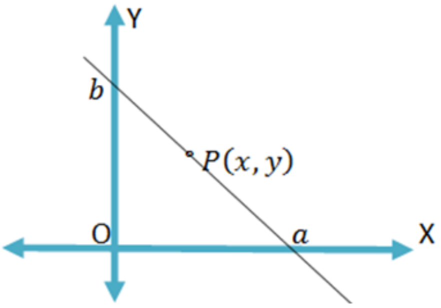


Fig. 4.10

Example 36. Find the equation of straight line which makes intercepts 2 and 5 on X-axis and Y-axis respectively.

Sol. Given that X-intercept is 2 and Y-intercept is 5

i.e. $a = 2$ and $b = 5$

We know that equation of straight line in Intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{2} + \frac{y}{5} = 1$$

$$\Rightarrow \quad \frac{5x + 2y}{10} = 1$$

$$\Rightarrow \quad 5x + 2y = 10$$

$$\Rightarrow \quad 5x + 2y - 10 = 0$$

which is the required equation of straight line.

Example 37. Find the equation of straight line which makes intercepts 3 and -15 on the axes.

Sol. Given that X-intercept is 3 and Y-intercept is -15

i.e. $a = 3$ and $b = -15$

We know that equation of straight line in Intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{3} + \frac{y}{-15} = 1$$

$$\Rightarrow \quad \frac{x}{3} - \frac{y}{15} = 1$$

$$\Rightarrow \quad \frac{5x - y}{15} = 1$$

$$\Rightarrow \quad 5x - y = 15$$

$$\Rightarrow \quad 5x - y - 15 = 0$$

which is the required equation of straight line.

Example 38. Find the equation of straight line which passes through $(1,-4)$ and makes intercepts on axes which are equal in magnitude and opposite in sign.

Sol. Let the intercepts on the axes are p and $-p$

i.e. $a = p$ and $b = -p$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{p}+\frac{y}{-p}=1$$

$$\Rightarrow \quad \frac{x}{p}-\frac{y}{p}=1$$

$$\Rightarrow \quad \frac{x-y}{p}=1$$

$$\Rightarrow \quad x-y=p \qquad (1)$$

Given that this line passes through $(1,-4)$.

Therefore put $x = 1$ and $y = -4$ in equation (1), we get

$$1-(-4)=p$$

$$\Rightarrow \quad p=5$$

Using this value in equation (1), we get

$$x-y=5$$

$$\Rightarrow \quad x-y-5=0$$

which is the required equation of straight line.

Example 39. Find the equation of straight line which passes through $(1,4)$ and sum of whose intercepts on axes is 10.

Sol. Let the intercepts on the axes are p and $10-p$

i.e. $a = p$ and $b = 10-p$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{p}+\frac{y}{10-p}=1$$

$$\Rightarrow \quad \frac{(10-p)x+py}{p(10-p)}=1$$

$$\Rightarrow \quad (10-p)x+py=p(10-p) \qquad (1)$$

Given that this line passes through $(1,4)$.

Therefore put $x = 1$ and $y = 4$ in equation (1), we get

$$(10-p)(1)+p(4)=p(10-p)$$

$$\Rightarrow \quad 10-p+4p=10p-p^2$$

$$\Rightarrow \quad p^2-7p+10=0$$

$\Rightarrow \quad p^2 - 5p - 2p + 10 = 0$

$\Rightarrow \quad p(p-5) - 2(p-5) = 0$

$\Rightarrow \quad (p-2)(p-5) = 0$

either $p-2=0$ *or* $p-5=0$

either $p=2$ *or* $p=5$

Put $p=2$ in equation (1), we get

$\Rightarrow \quad (10-2)x + 2y = 2(10-2)$

$\Rightarrow \quad 8x + 2y = 16$

$\Rightarrow \quad 4x + y = 8 \qquad (2)$

Put $p=5$ in equation (1), we get

$\Rightarrow \quad (10-5)x + 5y = 5(10-5)$

$\Rightarrow \quad 5x + 5y = 25$

$\Rightarrow \quad x + y = 5 \qquad (3)$

Equations (2) and (3) are the required equations of straight lines.

Symmetric Form of a Straight Line

If α is the inclination of a straight line L passing through the point (x_1, y_1), then the equation of the straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}.$$

Example 40. Find the equation of the straight line with inclination $45°$ and passing through the point $(\sqrt{3}, 1)$ by Symmetric form.

Sol. Given that the straight line passing through the point $(\sqrt{3}, 1)$ with inclination $45°$.

Here $x_1 = \sqrt{3}$, $y_1 = 1$ and $\alpha = 45°$. So, by Symmetric form, equation of straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{\cos 45°} = \frac{y - 1}{\sin 45°}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{y - 1}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{1} = \frac{y - 1}{1}$$

$$\Rightarrow \quad x - \sqrt{3} = y - 1$$

$$\Rightarrow \quad x - y - \sqrt{3} + 1 = 0$$

which is required equation of straight line.

Example 41. Find the equation of the straight line with inclination $30°$ and passing through the point $(2, 5)$ by Symmetric form.

Sol. Given that the straight line passing through the point $(2, 5)$ with inclination $30°$.

Here $x_1 = 2, y_1 = 5$ and $\alpha = 30°$. So, by Symmetric form, equation of straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$$

$$\Rightarrow \quad \frac{x - 2}{\cos 30°} = \frac{y - 5}{\sin 30°}$$

$$\Rightarrow \quad \frac{x - 2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{y - 5}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad \frac{x - 2}{\sqrt{3}} = \frac{y - 5}{1}$$

$$\Rightarrow \quad x - 2 = \sqrt{3}\,y - 5\sqrt{3}$$

$$\Rightarrow \quad x - \sqrt{3}\,y - 2 + 5\sqrt{3} = 0$$

which is required equation of straight line.

Equation of Straight Line in Normal form:

Let p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis (see Fig. 4.11). If (x, y) be moving point on the line, then equation of straight line is

$$\boldsymbol{p = x\,cos\alpha + y\,sin\alpha}$$

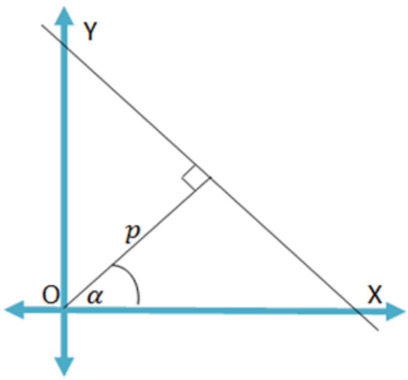


Fig. 4.11

Example 42. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 2 and the inclination of this perpendicular to the X-axis is 120°.

Sol. We know that equation of straight line in Normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (1)$$

where p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis.

Here $p = 2$ and $\alpha = 120^\circ$. Put these values in (1), we get

$$x\cos 120^\circ + y\sin 120^\circ = 2$$

$\Rightarrow \quad x\cos(180^\circ - 60^\circ) + y\sin(180^\circ - 60^\circ) = 2$

$\Rightarrow \quad -x\cos(60^\circ) + y\sin(60^\circ) = 2$

$\Rightarrow \quad -x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 2$

$\Rightarrow \quad \frac{-x + \sqrt{3}\,y}{2} = 2$

$\Rightarrow \quad -x + \sqrt{3}\,y = 4$

$\Rightarrow \quad -x + \sqrt{3}\,y - 4 = 0$

which is the required equation of straight line.

Example 43. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 7 and the inclination of this perpendicular to the X-axis is 45°.

Sol. We know that equation of straight line in Normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (1)$$

where p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis.

Here $p = 7$ and $\alpha = 45^\circ$. Put these values in (1), we get

$$x\cos 45^\circ + y\sin 45^\circ = 7$$

$\Rightarrow \quad x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = 7$

$\Rightarrow \quad x + y = 7\sqrt{2}$

$\Rightarrow \quad x + y - 7\sqrt{2} = 0$

which is the required equation of straight line.

<u>Intersection of Two Lines</u>:

Let $\quad a_1x + b_1y + c_1 = 0 \qquad (1)$

and $\quad a_2x + b_2y + c_2 = 0 \qquad (2)$

be the two straight lines.

Their point of intersection can be obtained by:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$(provided\ that\ a_1b_2 - a_2b_1 \neq 0)$$

$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad and \quad y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Hence the point of intersection of given straight lines is

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

In case when $a_1b_2 - a_2b_1 = 0$, then the above coordinates have no meaning. In this case the lines do not intersect, but are parallel.

Example 44. Check whether the following straight lines are intersecting lines and if they are intersecting lines find their point of intersection:

(i) $2x + 5y = -4$ and $x + 6y = 5$

(ii) $3x - 4y = -5$ and $6x - 8y = -10$

Sol. (i) Given equations are $2x + 5y = -4$ and $x + 6y = 5$

$\Rightarrow$ $2x + 5y + 4 = 0$ and $x + 6y - 5 = 0$

Comparing these equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

We have, $a_1 = 2,\ b_1 = 5,\ c_1 = 4,\ a_2 = 1,\ b_2 = 6\ and\ c_2 = -5.$

Now, $a_1b_2 - a_2b_1 = 2(6) - 1(5) = 12 - 5 = 7 \neq 0.$

Therefore the given lines are intersecting lines.

Also their point of intersection is given by

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

$$= \left(\frac{5(-5) - 6(4)}{2(6) - 1(5)}, \frac{1(4) - 2(-5)}{2(6) - 1(5)}\right)$$

$$= \left(\frac{-25 - 24}{12 - 5}, \frac{4 + 10}{12 - 5}\right)$$

$$= \left(\frac{-49}{7}, \frac{14}{7}\right)$$

$$= (-7, 2)$$

which is required point.

(ii) Given equations are $3x - 4y = -5$ and $6x - 8y = -10$

$\Rightarrow$ $3x - 4y + 5 = 0$ and $6x - 8y + 10 = 0$

Comparing these equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

We have, $a_1 = 3,\ b_1 = -4,\ c_1 = 5,\ a_2 = 6,\ b_2 = -8\ and\ c_2 = 10.$

Now, $a_1b_2 - a_2b_1 = 3(-8) - 6(-4) = -24 + 24 = 0.$

Therefore the given lines are not intersecting lines.

Concurrency of Lines:

Three or more than three straight lines are said to be concurrent if these are intersecting at the same point. The point of intersection of these lines is called point of concurrency.

Condition of Concurrency of Three Lines:

Let $a_1x + b_1y + c_1 = 0$ (1)

$a_2x + b_2y + c_2 = 0$ (2)

and $a_3x + b_3y + c_3 = 0$ (3)

+be the three straight lines.

These three straight lines will be concurrent if the point of intersection of any two lines satisfies the third line.

The point of intersection of (1) and (2) is

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

Using this point in equation (3), we have

$$a_3\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right) + b_3\left(\frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right) + c_3 = 0$$

$\Rightarrow \quad a_3(b_1c_2 - b_2c_1) + b_3(a_2c_1 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

This is same as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Example 45. Show that the following lines are concurrent and also find their find of concurrency:

$$x - y + 6 = 0$$

$$2x + y - 5 = 0$$

and $\quad -x - 2y + 11 = 0$

Sol. Given equations are

$$x - y + 6 = 0, \quad (1)$$

$$2x + y - 5 = 0 \quad (2)$$

and $\quad -x - 2y + 11 = 0 \quad (3)$

Comparing these equations with

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$

We have, $a_1 = 1,\ b_1 = -1,\ c_1 = 6,\ a_2 = 2,\ b_2 = 1,\ c_2 = -5,$

$a_3 = -1,\ b_3 = -2\ and\ c_3 = 11$

Now, $\quad a_1(b_2c_1 - b_1c_2) + b_1(a_1c_2 - a_2c_1) + c_1(a_2b_1 - a_1b_2)$

$= 1\,[1(6) - (-1)(-5)] - 1\,[1(-5) - 2(6)] + 6\,[2(-1) - 1(1)]$

$= 1\,[6 - 5] - 1\,[-5 - 12] + 6\,[-2 - 1]$

$= 1 + 17 - 18 = 0$

Hence the given lines are concurrent.

To find the point of concurrency, let us solve equation (1) and equation (2).

Intersection point of equation (1) and equation (2) is given by

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

$$= \left(\frac{-1(-5) - 1(6)}{1(1) - 2(-1)}, \frac{2(6) - 1(-5)}{1(1) - 2(-1)}\right)$$

$= \quad \left(\frac{5-6}{1+2}, \frac{12+5}{1+2}\right)$

$= \quad \left(\frac{-1}{3}, \frac{17}{3}\right)$

which is the required point of concurrency.

Example 46. Find the value of k, if the following lines are concurrent:

$x - 2y + 1 = 0$

$2x - 5y + 3 = 0$

and $\quad 5x + 9y + k = 0$

Sol. Given equations are

$x - 2y + 1 = 0$, (1)

$2x - 5y + 3 = 0$ (2)

and $\quad 5x + 9y + k = 0$ (3)

Comparing these equations with

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$

We have, $a_1 = 1,\ b_1 = -2,\ c_1 = 1,\ a_2 = 2,\ b_2 = -5,\ c_2 = 3,$

$a_3 = 5,\ b_3 = 9\ and\ c_3 = k$

It is given that the given lines are concurrent. Therefore by condition of concurrency, we have

$a_3(b_1c_2 - b_2c_1) + b_3(a_2c_1 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

$\Rightarrow \quad 5\,[-2(3) - (-5)(1)] + 9\,[2(1) - 1(3)] + k\,[1(-5) - 2(-2)] = 0$

$\Rightarrow \quad 5\,[-6 + 5] + 9\,[2 - 3] + k\,[-5 + 4] = 0$

$\Rightarrow \quad -5 - 9 - k = 0$

$\Rightarrow \quad k = -14$

which is required value of k.

<u>Angle Between Two Straight Lines</u>: Two intersecting lines always intersects at two angles in which one angle is acute angle and another angle is obtuse angle. The sum of both the angles is 180° i.e. they are supplementary to each other. For example, if one angle between intersecting lines is 60° then another angle is $180^\circ - 60^\circ = 120^\circ$.

Generally, we take acute angle as the angle between the lines (see Fig. 4.12).

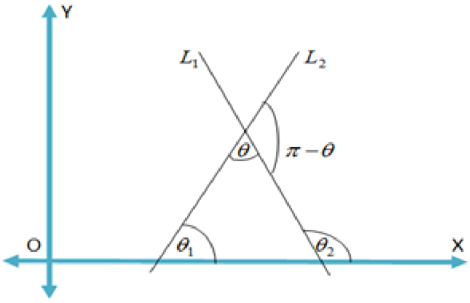


Fig. 4.12

Let L_1 & L_2 be straight lines and m_1 & m_2 be their slopes respectively.

Let θ_1 & θ_2 be the angles which L_1 & L_2 make with positive X-axis respectively.

Therefore $m_1 = \tan(\theta_1)$ & $m_2 = \tan(\theta_2)$.

Let θ be the acute angle between lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad or \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

Example 47. Find the acute angle between the lines whose slopes are 1 and 0.

Sol. Given that slopes of lines are 1 and 0.

Let $m_1 = 1$ and $m_2 = 0$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1-0}{1+(1)(0)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1}{1+0}\right|$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

which is the required acute angle.

Example 48. Find the acute angle between the lines whose slopes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Sol. Given that slopes of lines are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Let $m_1 = 2 + \sqrt{3}$ and $m_2 = 2 - \sqrt{3}$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{(2+\sqrt{3})-(2-\sqrt{3})}{1+(2+\sqrt{3})(2-\sqrt{3})}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{2\sqrt{3}}{2}\right|$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

which is the required acute angle.

Example 49. Find the obtuse angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Sol. Given that slopes of lines are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Let $m_1 = \sqrt{3}$ and $m_2 = \frac{1}{\sqrt{3}}$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \quad \tan\theta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{3-1}{\sqrt{3}}}{1+1}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{2}{2\sqrt{3}}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan\left(30^\circ\right)$$

$$\Rightarrow \quad \theta = 30^\circ$$

Therefore, $180^\circ - \theta$ is the obtuse angle between the lines.

i.e. $180^\circ - 30^\circ = 150^\circ$ is the obtuse angle between the lines.

Example 50. Find the angle between the lines whose slopes are -3 and 5.

Sol. Given that slopes of lines are -3 and 5.

Let $m_1 = -3$ and $m_2 = 5$.

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \quad \tan\theta = \left|\frac{-3-5}{1+(-3)(5)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-8}{1-15}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-8}{-14}\right|$$

$$\Rightarrow \qquad \tan\theta = \frac{4}{7}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{4}{7}\right)$$

which is the required angle.

Example 51. Find the angle between the lines joining the points $(0,0)$, $(2,3)$ and $(2,-2)$, $(3,5)$.

Sol. Let m_1 be the slope of the line joining $(0,0)$ and $(2,3)$ and m_2 be the slope of the line joining $(2,-2)$ and $(3,5)$.

$$\Rightarrow \qquad m_1 = \frac{3-0}{2-0} = \frac{3}{2}$$

and $$m_2 = \frac{5-(-2)}{3-2} = \frac{7}{1} = 7.$$

Also let θ be the angle between lines.

Therefore, $$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{\frac{3}{2} - 7}{1 + \left(\frac{3}{2}\right)(7)}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{\frac{3-14}{2}}{\frac{2+21}{2}}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{\frac{-11}{2}}{\frac{23}{2}}\right|$$

$$\Rightarrow \qquad \tan\theta = \frac{11}{23}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{11}{23}\right)$$

which is the required angle.

Example 52. Find the angle between the lines joining the points $(6,-5)$, $(-2,1)$ and $(0,3)$, $(-8,6)$.

Sol. Let m_1 be the slope of the line joining $(6,-5)$ and $(-2,1)$ and m_2 be the slope of the line joining $(0,3)$ and $(-8,6)$.

$$\Rightarrow \qquad m_1 = \frac{1-(-5)}{-2-6} = -\frac{6}{8} = -\frac{3}{4}$$

and $$m_2 = \frac{6-3}{-8-0} = -\frac{3}{8}.$$

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \qquad \tan\theta = \left|\frac{-\frac{3}{4} - \left(-\frac{3}{8}\right)}{1 + \left(-\frac{3}{4}\right)\left(-\frac{3}{8}\right)}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{-\frac{3}{4} + \frac{3}{8}}{1 + \frac{9}{32}}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{\frac{-6+3}{8}}{\frac{32+9}{32}}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{\frac{-3}{8}}{\frac{41}{32}}\right|$$

$$\Rightarrow \qquad \tan\theta = \left|\frac{-3}{8} \times \frac{32}{41}\right|$$

$$\Rightarrow \qquad \tan\theta = \frac{12}{41}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{12}{41}\right)$$

which is the required angle.

Example 53. Find the angle between the pair of straight lines

$(-2 + \sqrt{3})x + y + 9 = 0$ and $(2 + \sqrt{3})x - y + 20 = 0$.

Sol. Given that equations of lines are

$$(-2 + \sqrt{3})x + y + 9 = 0 \qquad (1)$$

and $$(2 + \sqrt{3})x - y + 20 = 0 \qquad (2)$$

Let m_1 be the slope of the line (1) and m_2 be the slope of the line (2).

$$\Rightarrow \qquad m_1 = -\frac{-2 + \sqrt{3}}{1} = 2 - \sqrt{3}$$

and $$m_2 = -\frac{2 + \sqrt{3}}{-1} = 2 + \sqrt{3}.$$

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \qquad \tan\theta = \left| \frac{(2-\sqrt{3})-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-2\sqrt{3}}{2} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| -\sqrt{3} \right|$$

$$\Rightarrow \qquad \tan\theta = \sqrt{3}$$

$$\Rightarrow \qquad \tan\theta = \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \qquad \theta = \frac{\pi}{3}$$

which is the required angle.

Example 54. Find the angle between the pair of straight lines

$x + \sqrt{3}\,y - 8 = 0$ and $x - \sqrt{3}\,y + 2 = 0$.

Sol. Given that equations of lines are

$$x + \sqrt{3}\,y - 8 = 0 \qquad (1)$$

and $$x - \sqrt{3}\,y + 2 = 0 \qquad (2)$$

Let m_1 be the slope of the line (1) and m_2 be the slope of the line (2).

$$\Rightarrow \qquad m_1 = -\frac{1}{\sqrt{3}}$$

and $$m_2 = -\frac{1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| -\frac{3}{\sqrt{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{3\sqrt{3}}{3}$$

$$\Rightarrow \qquad \tan\theta = \sqrt{3}$$

$$\Rightarrow \qquad \tan\theta = \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \qquad \theta = \frac{\pi}{3}$$

which is the required angle.

Example 55. Find the equations of straight lines making an angle $45°$ with the line $6x + 5y - 1 = 0$ and passing through the point $(2, -1)$.

Sol. Given that equations of line is

$$6x + 5y - 1 = 0 \qquad (1)$$

Let m_1 be the slope of the line (1).

$$\Rightarrow \qquad m_1 = -\frac{6}{5}.$$

Let m_2 be the slope of required line.

Therefore, $\tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1\, m_2} \right|$

$$\Rightarrow \qquad 1 = \left| \frac{-\frac{6}{5} - m_2}{1 + \left(-\frac{6}{5}\right)(m_2)} \right|$$

$$\Rightarrow \qquad 1 = \left| \frac{\frac{-6-5m_2}{5}}{\frac{5-6m_2}{5}} \right|$$

$$\Rightarrow \qquad 1 = \left| \frac{-6-5m_2}{5-6m_2} \right|$$

$$\Rightarrow \qquad 1=\pm\left(\frac{-6-5m_2}{5-6m_2}\right)$$

$$\Rightarrow \qquad 5-6m_2=\pm(-6-5m_2)$$

Taking positive sign, we get

$$5-6m_2=+(-6-5m_2)$$

$$\Rightarrow \qquad 5-6m_2=-6-5m_2$$

$$\Rightarrow \qquad -m_2=-11$$

$$\Rightarrow \qquad m_2=11$$

So, equation of line passing through $(2,-1)$ with slope 11 is

$$y+1=11(x-2)$$

$$\Rightarrow \qquad y+1=11x-22$$

$$\Rightarrow \qquad 11x-y-23=0 \qquad (2)$$

Now taking negative sign, we get

$$5-6m_2=-(-6-5m_2)$$

$$\Rightarrow \qquad 5-6m_2=6+5m_2$$

$$\Rightarrow \qquad -11m_2=1$$

$$\Rightarrow \qquad m_2=-\frac{1}{11}$$

So, equation of line passing through $(2,-1)$ with slope $-\frac{1}{11}$ is

$$y+1=-\frac{1}{11}(x-2)$$

$$\Rightarrow \quad 11y+11=-x+2$$

$$\Rightarrow \quad x+11y+9=0 \qquad (3)$$

Equations (2) and (3) are required equations of straight lines.

Parallel And Perpendicular Lines:

Parallel Lines: Two lines are said to be parallel if they never intersect.

Perpendicular Lines: Two lines are said to be perpendicular if they intersect at right angle($i.e.\ 90°$).

Note I: Let L_1 and L_2 be two straight lines with slopes m_1 and m_2 respectively, then

(i) L_1 is parallel to L_2 ($i.e. L_1||L_2$) if and only if $m_1=m_2$.

(ii) L_1 is perpendicular to L_2 ($i.e. L_1 \perp L_2$) if and only if their slopes are negative-reciprocals to each other $i.e.$ $m_1=-\frac{1}{m_2}$ or $m_2=-\frac{1}{m_1}$.

Note II: Let $a_1x + b_1y + c_1 = 0$ (1)

and $a_2x + b_2y + c_2 = 0$ (2)

be the two straight lines.

(i) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, then the straight lines (1) and (2) are said to be parallel lines.

(ii) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then the straight lines (1) and (2) are said to be coincident lines.

(iii) If $a_1 a_2 + b_1 b_2 = 0$, then the straight lines (1) and (2) are said to be perpendicular lines.

Example 56. Check that the following pair of straight lines are parallel or perpendicular or neither:

(i) $2x + 3y = 9$ and $4x + 6y = 12$

(ii) $3x - 5y = 7$ and $10x + 6y = 12$

(iii) $4x + 4y = 18$ and $3x - 2y = 4$

(iv) $y = 2x - 3$ and $y = 2x + 1$

(v) $3y = 7x + 2$ and $7y = -3x - 5$

Sol. (i) The given equations of straight lines are

$$2x + 3y = 9 \quad \text{and} \quad 4x + 6y = 12$$

Comparing these equations with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we have

$$a_1 = 2, \quad b_1 = 3, \quad c_1 = -9, \quad a_2 = 4, \quad b_2 = 6 \ and \ c_2 = -12$$

Now

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-12}{-9} = \frac{4}{3}$$

From above it is clear that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Hence the given straight lines are parallel lines.

Alternate Method: The given equations of straight lines are

$$2x + 3y = 9 \tag{1}$$

and $$4x + 6y = 12 \tag{2}$$

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$$m_1 = -\frac{2}{3} \quad \text{and} \quad m_2 = -\frac{4}{6} = -\frac{2}{3}$$

From above it is clear that $m_1 = m_2$.

Hence the given straight lines are parallel lines.

(ii) The given equations of straight lines are

$$3x - 5y = 7 \quad \text{and} \quad 10x + 6y = 12$$

Comparing these equations with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we have

$$a_1 = 3, \quad b_1 = -5, \quad c_1 = -7, \quad a_2 = 10, \quad b_2 = 6 \ and \ c_2 = -12$$

Now

$$\frac{a_1}{a_2} = \frac{3}{10},$$

$$\frac{b_1}{b_2} = \frac{-5}{6}$$

and $$\frac{c_1}{c_2} = \frac{-7}{-12} = \frac{7}{12}$$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 3(10) - 5(6) = 30 - 30 = 0$

Hence the given straight lines are perpendicular lines.

Alternate Method: The given equations of straight lines are

$$3x - 5y = 7 \qquad (1)$$

and $$10x + 6y = 12 \qquad (2)$$

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$$m_1 = -\frac{3}{-5} = \frac{3}{5} \quad \text{and} \quad m_2 = -\frac{10}{6} = -\frac{5}{3}$$

From above it is clear that $m_1 . m_2 = \frac{3}{5} . \left(-\frac{5}{3}\right) = -1.$

Hence the given straight lines are perpendicular lines.

(iii) The given equations of straight lines are

$$4x + 4y = 18 \text{ and } \quad 3x - 2y = 4$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$$a_1 = 4, \quad b_1 = 4, \quad c_1 = -18, \quad a_2 = 3, \quad b_2 = -2 \ and \ c_2 = -4$$

Now

$$\frac{a_1}{a_2} = \frac{4}{3},$$

$$\frac{b_1}{b_2} = \frac{4}{-2} = -2$$

and $$\frac{c_1}{c_2} = \frac{-18}{-4} = \frac{9}{2}$$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 4(3) + 4(-2) = 12 - 8 = 4$

Hence the given straight lines are neither parallel nor perpendicular.

(iv) The given equations of straight lines are

$$y = 2x - 3 \qquad (1)$$

and $$y = 2x + 1 \qquad (2)$$

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$$m_1 = 2 \quad \text{and} \quad m_2 = 2 \qquad \text{(by slope-intercept form)}$$

From above it is clear that $m_1 = m_2$.

Hence the given straight lines are parallel lines.

(v) The given equations of straight lines are

$$3y = 7x + 2 \quad \text{and} \quad 7y = -3x - 5$$

i.e. $7x - 3y + 2 = 0$ and $-3x - 7y - 5 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 7,\quad b_1 = -3,\quad c_1 = 2,\quad a_2 = -3,\quad b_2 = -7$ and $c_2 = -5$

Now

$$\frac{a_1}{a_2} = \frac{7}{-3} = -\frac{7}{3},$$

$$\frac{b_1}{b_2} = \frac{-3}{-7} = \frac{3}{7}$$

and $$\frac{c_1}{c_2} = \frac{2}{-5} = -\frac{2}{5}$$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 7(-3) - 3(-7) = -21 + 21 = 0$

Hence the given straight lines are perpendicular lines.

The Perpendicular Distance from a Point to a Straight Line

Let $P_1(x_1\,,\ y_1)$ be any point and $ax + by + c = 0$ be any straight line. To find the distance from the point P_1 to the given straight line, draw the perpendicular P_1P_2 from P_1 to the given straight line (as shown in Fig. 4.13). Let the coordinates of P_2 are $(x_2\,,\ y_2)$.

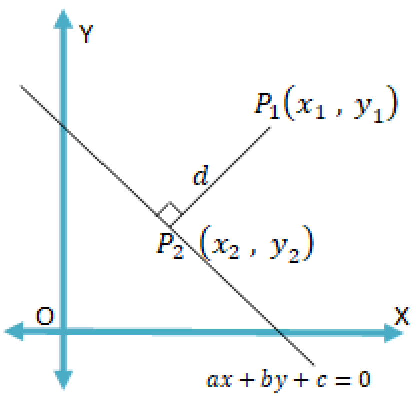


Fig. 4.13

Let d is the distance between the points P_1 and P_2. So by distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

There is requirement to find the coordinates of point P_2 .

Since $P_2(x_2\,,\ y_2)$ lies on $ax + by + c = 0$, therefore we have

$$ax_2 + by_2 + c = 0 \qquad (2)$$

The slope of $ax + by + c = 0$ is $-\frac{a}{b}$.

Let m is the slope of perpendicular $\overline{P_1P_2}$, so $m = \frac{-1}{\left(-\frac{a}{b}\right)} = \frac{b}{a}$.

Also the slope of a straight line passing through $(x_1\,,\ y_1)$ and $(x_2\,,\ y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$.

Therefore, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$

$\Rightarrow \quad ay_2 - ay_1 = bx_2 - bx_1$

$\Rightarrow \quad bx_2 - ay_2 + ay_1 - bx_1 = 0$ (3)

Solving (2) and (3), we have

$$\frac{x_2}{b(ay_1 - bx_1) + ac} = \frac{-y_2}{a(ay_1 - bx_1) - bc} = \frac{1}{-a^2 - b^2}$$

$\Rightarrow \quad x_2 = \dfrac{b^2x_1 - aby_1 - ac}{a^2 + b^2}$

and $\quad y_2 = \dfrac{a^2y_1 - abx_1 - bc}{a^2 + b^2}$

Using the values of x_2 and y_2 in equation (1), we have

$$d = \sqrt{\left(\frac{b^2x_1 - aby_1 - ac}{a^2+b^2} - x_1\right)^2 + \left(\frac{a^2y_1 - abx_1 - bc}{a^2+b^2} - y_1\right)^2}$$

After simplifying, we have

$$d = \frac{|\,ax_1 + by_1 + c\,|}{\sqrt{a^2+b^2}}$$

which is the required distance from point $P_1(x_1, y_1)$ to the straight line $ax + by + c = 0$.

Example 57. Find the distance from the point $(-5, 2)$ to the straight line $4x - 2y - 3 = 0$.

Sol. Let d is the distance from the point $(-5, 2)$ to the straight line $4x - 2y - 3 = 0$.

Therefore, $d = \frac{|\,4(-5) - 2(2) - 3\,|}{\sqrt{(4)^2 + (-2)^2}}$

$\Rightarrow \quad d = \frac{|-20 - 4 - 3|}{\sqrt{16+4}}$

$\Rightarrow \quad d = \frac{|-27|}{\sqrt{20}}$

$\Rightarrow \quad d = \frac{27}{2\sqrt{5}}$

which is the required distance.

Example 58. Find the distance from the point $(3, 5)$ to the straight line $3x + 4y + 6 = 0$.

Sol. Let d is the distance from the point $(3, 5)$ to the straight line $3x + 4y + 6 = 0$.

Therefore, $d = \frac{|\,3(3) + 4(5) + 6\,|}{\sqrt{(3)^2 + (4)^2}}$

$\Rightarrow \quad d = \frac{|\,9+20+6\,|}{\sqrt{9+16}}$

$\Rightarrow \quad d = \frac{|\,35\,|}{\sqrt{25}}$

$\Rightarrow \quad d = \frac{35}{5}$

$\Rightarrow \quad d = 7$

which is the required distance.

Example 59. If p is the perpendicular distance from origin to the straight line whose intercepts are a and b on x-axis and y-axis respectively, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Sol. The equation of straight line whose intercepts are a and b on is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad bx + ay - ab = 0 \qquad (1)$$

Since p is the perpendicular distance from origin to the straight line (1).

Therefore, $p = \frac{|\, b\,(0) + a\,(0) - ab\,|}{\sqrt{(b)^2 + (a)^2}}$

$$\Rightarrow \quad p = \frac{|-ab\,|}{\sqrt{(b)^2 + (a)^2}}$$

$$\Rightarrow \quad p^2 = \frac{a^2\,b^2}{a^2 + b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence proved.

General Equation of Straight Line:

The general equation of straight line in variables x and y is an equation of the form

$$ax + by + c = 0 \qquad (1)$$

where a, b and c are given real numbers and a and b are not both zero simultaneously.

Reduction of General form $ax + by + c = 0$ to other forms:

(i) Reduction to Slope–Intercept Form:

The general form of straight line is $ax + by + c = 0$

$$\Rightarrow \quad by = -ax - c$$

$$\Rightarrow \quad y = -\frac{a}{b}x - \frac{c}{b}$$

which is of the form $y = mx + c$.

(ii) Reduction to Intercept Form:

The general form of straight line is $ax + by + c = 0$

$$\Rightarrow \quad ax + by = -c$$

$$\Rightarrow \quad -\frac{a}{c}x - \frac{b}{c}y = 1$$

$$\Rightarrow \quad \frac{x}{\left(-\frac{c}{a}\right)} + \frac{y}{\left(-\frac{c}{b}\right)} = 1$$

which is of the form $\frac{x}{A} + \frac{y}{B} = 1$.

(iii) Reduction to Normal Form:

The general form of straight line is

$$ax + by + c = 0 \qquad (1)$$

Comparing this equation with the Normal form

$$\cos\alpha\, x + \sin\alpha\, y - p = 0 \qquad (2)$$

we have, $\frac{a}{\cos\alpha} = \frac{b}{\sin\alpha} = \frac{c}{-p} = k$

because the coefficient of (1) and (2) are proportional.

So $a = k\cos\alpha$, $b = k\sin\alpha$ and $c = -pk$

$\Rightarrow \quad a^2 + b^2 = k^2(cos^2\alpha + sin^2\alpha)$

$\Rightarrow \quad a^2 + b^2 = k^2$

$\Rightarrow \quad a^2 + b^2 = k^2$

$\Rightarrow \quad k = \pm\sqrt{a^2 + b^2}$

Therefore, $\quad p = -\frac{c}{k}$

$$\Rightarrow \quad p = -\left(\pm\frac{c}{\sqrt{a^2 + b^2}}\right)$$

As p must be positive, so the signs of c and $\sqrt{a^2 + b^2}$ must be opposite.

Case I: If c is positive, then $k = -\sqrt{a^2 + b^2}$

Then from $\cos\alpha = \frac{a}{k}$, $\sin\alpha = \frac{b}{k}$ and $p = -\frac{c}{k}$, we have

$$\cos\alpha = -\frac{a}{\sqrt{a^2 + b^2}}, \qquad \sin\alpha = -\frac{b}{\sqrt{a^2 + b^2}} \quad and \quad p = \frac{c}{\sqrt{a^2 + b^2}}$$

Using these values in equation (2), we have

$$-\frac{a}{\sqrt{a^2 + b^2}}\, x - \frac{b}{\sqrt{a^2 + b^2}}\, y - \frac{c}{\sqrt{a^2 + b^2}} = 0$$

$$\Rightarrow \quad \frac{a}{\sqrt{a^2 + b^2}}\, x + \frac{b}{\sqrt{a^2 + b^2}}\, y + \frac{c}{\sqrt{a^2 + b^2}} = 0$$

which is the required Normal form.

Case II: If c is negative, then $k = \sqrt{a^2 + b^2}$

Then from $\cos\alpha = \frac{a}{k}$, $\sin\alpha = \frac{b}{k}$ and $p = -\frac{c}{k}$, we have

$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}, \qquad \sin\alpha = \frac{b}{\sqrt{a^2+b^2}} \quad and \quad p = -\frac{c}{\sqrt{a^2+b^2}}$$

Using these values in equation (2), we have

$$\frac{a}{\sqrt{a^2+b^2}}\,x + \frac{b}{\sqrt{a^2+b^2}}\,y + \frac{c}{\sqrt{a^2+b^2}} = 0$$

which is the required Normal form.

Example 60. Reduce the equation $3x + 4y = 10$ to the

(i) Slope-intercept from

(ii) Intercept form

(iii) Normal form

Sol. (i) Given equation is $3x + 4y = 10$

$\Rightarrow \quad 4y = -3x + 10$

$\Rightarrow \quad y = -\frac{3}{4}x + \frac{10}{4}$

$\Rightarrow \quad y = -\frac{3}{4}x + \frac{5}{2}$

which is the slope-intercept form with slope $-\frac{3}{4}$ and $y - intercept = \frac{5}{2}$.

(ii) Given equation is $3x + 4y = 10$

$\Rightarrow \quad \frac{3}{10}x + \frac{4}{10}y = 1$

$\Rightarrow \quad \frac{3}{10}x + \frac{2}{5}y = 1$

$\Rightarrow \quad \frac{x}{\left(\frac{10}{3}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$

which is the Intercept form with $x - intercept = \frac{10}{3}$ and $y - intercept = \frac{5}{2}$.

(iii) Given equation is $3x + 4y = 10$

$\Rightarrow \quad 3x + 4y - 10 = 0$

Here $a = 3$, $b = 4$ and $c = -10$.

Therefore, $k = \sqrt{a^2+b^2}$

$\Rightarrow \quad k = \sqrt{3^2+4^2}$

$\Rightarrow \quad k = \sqrt{9+16}$

$\Rightarrow \quad k = \sqrt{25}$

$\Rightarrow \quad k = 5$

Dividing the given equation by k, i.e., 5 , we have

$\Rightarrow \quad \frac{3}{5}x + \frac{4}{5}y - \frac{10}{5} = 0$

$\Rightarrow \quad \frac{3}{5}x + \frac{4}{5}y = 2$

which is the required Normal form with $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$ and $p = 2$.

<u>To find α</u>: $\frac{\sin\alpha}{\cos\alpha} = \frac{4}{3}$

$\Rightarrow$ $\tan\alpha = \frac{4}{3}$

$\Rightarrow$ $\alpha = tan^{-1}\left(\frac{4}{3}\right)$

Hence the given straight line is at a distance of 2 unit from the origin and is perpendicular from the origin on the line with angle of inclination $= tan^{-1}\left(\frac{4}{3}\right)$.

EXERCISE-II

1. The slope of Y-axis is:
   (a) infinity (b) 0 (c) ½ (d) 1
2. If two lines are intersecting at an angle of 60°, then the other angle between these two lines is:
   (a) 120° (b) 60° (c) 90° (d) 180°
3. If the equation of straight line is $ax + by + c = 0$, then slope of straight line is:
   (a) $-\frac{b}{a}$ (b) $-\frac{a}{b}$ (c) $\frac{b}{a}$ (d) c
4. The equation of a straight line passing through (x_1, y_1) and having slope m is:
   (a) $y - y_1 = m(x - x_1)$ (b) $x - x_1 = m(y - y_1)$
   (c) $y - y_1 = -m(x - x_1)$ (d) None of these
5. Find the equation of straight line passing through the point $(-5\,, 4)$ and having slope equal to -2.
6. Find the equations of straight lines which passing through the following pairs of points:
   (i) $(-11, -5), (-3, 10)$ (ii) $(0,0), (10, -12)$
7. Find the equation of straight line which makes an angle 60° with x-axis and cuts on intercepts 5 on y-axis above the x-axis.
8. Find the equation of straight line whose intercepts on X-axis and Y-axis are 3 and -9 respectively.
9. Find the equation of straight line which passes through $(2,3)$ and makes intercepts on axes which are equal in magnitude and are of same sign.
10. Find the equation of straight line which passes through $(2,4)$ and sum of whose intercepts on axes is 15.
11. Find the equation of straight line with inclination $45°$ and passing through the point $(2\,, \sqrt{2})$ by Symmetric form.
12. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 10 and the inclination of this perpendicular to the x-axis is 60°.
13. Find the angle between the lines joining the points $(0,0), (4,6)$ and $(1, -1)$, $(6,10)$.
14. Find the angle between the pair of straight lines
    $(-4 + \sqrt{3})x + y + 9 = 0$ and $(4 + \sqrt{3})x - y + 10 = 0$
15. Find the equation of straight line making an angle 60° with the line

$6x + 5y - 1 = 0$ and passing through the point $(1, -1)$.

16. Check whether the following straight lines are intersecting lines and if they are intersecting lines find their point of intersection:
    (i) $-x + 2y = 4$ and $2x + 6y = -1$
    (ii) $2x - y = -3$ and $4x - 2y = -6$
17. Show that the following lines are concurrent and also find their find of concurrency:
    $2x + 5y - 1 = 0$
    $x - 3y - 6 = 0$
    and $x + 5y + 2 = 0$
18. Check that the following pair of straight lines are parallel or perpendicular or neither:
    (i) $x + 2y = -5$ and $2x + 4y = 12$
    (ii) $3x + 4y = 2$ and $8x - 6y = 5$
    (iii) $y = -5x + 1$ and $5y = x + 10$
    (iv) $y = -5x - 3$ and $y = -5x + 7$
    (v) $4x - 3y = 10$ and $5x - y = 3$
19. Find the distance from the point $(1, 1)$ to the straight line $12x + 5y + 9 = 0$.
20. Find the distance from the point $(2, 3)$ to the straight line $4y = 3x + 1$.
21. Reduce the equation $4x - 3y = 5$ to the
    (i) Slope-intercept from
    (ii) Intercept form
    (iii) Normal form

ANSWERS

1. (a) 2. (a)

3. (b) 4.(a)

5. $2x + y + 6 = 0$

6. (i) $15x - 8y + 125 = 0$ (ii) $6x + 5y = 0$

7. $y = \sqrt{3}x + 5$ 8. $3x - y - 9 = 0$

9. $x + y = 5$ 10. $\frac{x}{10} + \frac{y}{5} = 1$ and $\frac{x}{3} + \frac{y}{12} = 1$

11. $x - y - 2 + \sqrt{2} = 0$ 12. $\frac{x}{2} + \frac{\sqrt{3}}{2}y = 10$

13. $tan^{-1}\left(\frac{7}{43}\right)$ 14. $tan^{-1}\left(\frac{\sqrt{3}}{7}\right)$

15. $y + 1 = \left(\frac{5\sqrt{3}-6}{6\sqrt{3}+5}\right)(x - 1)$ and $y + 1 = \left(\frac{5\sqrt{3}+6}{6\sqrt{3}-5}\right)(x - 1)$

16. (i) The given lines are intersecting lines and their point of intersection is $\left(-\frac{13}{5}, \frac{7}{10}\right)$.

(ii) The given lines are not intersecting lines.

17. Point of concurrency is $(3,-1)$.

18. (i) Parallel Lines (ii) Perpendicular Lines

(iii) Perpendicular Lines (iv) Parallel Lines

(v) Neither Parallel nor Perpendicular

19. 2 units

20. 1 unit

21. (i) $y = \frac{4}{3}x + \frac{5}{3}$, having slope $\frac{4}{3}$ and $y - intercept = \frac{5}{3}$

(ii) $\frac{x}{\left(\frac{5}{4}\right)} + \frac{y}{\left(-\frac{5}{3}\right)} = 1$, having $x - intercept = \frac{5}{4}$ and $y - intercept = -\frac{5}{3}$

(iii) $\frac{4}{5}x - \frac{3}{5}y = 1$, $\cos\alpha = \frac{4}{5}$, $\sin\alpha = -\frac{3}{5}$ and $p = 1$

UNIT V
Geometry of Circle and Software

Learning Objectives

- Students will be able to illustrate a circle and the terms related to it: center, radius and diameter.
- Students will be able to determine the equations of circles in various forms.
- Students will be able to learn about the basic fundamentals of MATLAB/ Scilab and to use these languages for mathematical calculations with MATLAB/ Scilab software.

5.1 CIRCLE

Circle: Circle is the locus of a point which moves in a plane such that its distance from a fixed point always remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

In Fig. 5.1, $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be the moving point on the circumference of the circle.

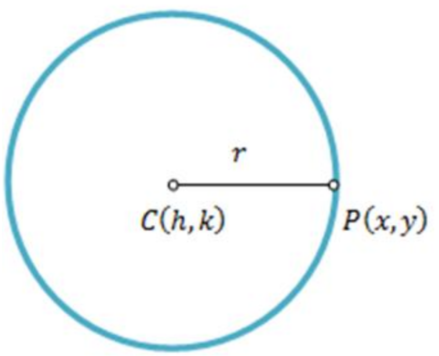


Fig. 5.1

Standard Equation of the Circle: Let $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be any point on the circle, then equation of circle is

$$(x-h)^2+(y-k)^2=r^2 \quad (1)$$

which is known as standard equation of circle. This is also known as central form of equation of circle.

Some Particular Cases:

Let $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be any point on the circle:

(i) When the centre of the circle coincides with the origin (see Fig. 5.2) i.e. $h=k=0$:

Thus equation (1) becomes:

$$\Rightarrow \quad (x-0)^2+(y-0)^2=r^2$$

$$\Rightarrow \quad x^2+y^2=r^2$$

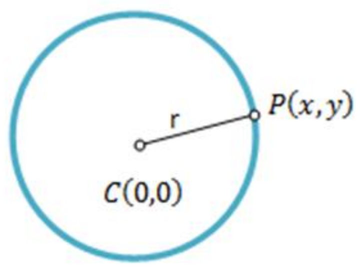


Fig. 5.2

(ii) When the circle passes through the origin (see Fig. 5.3):

Let CR be the perpendicular from the centre on X-axis. Therefore,

$$OR^2+CR^2=OC^2$$

$$\Rightarrow \quad (h-0)^2+(k-0)^2=r^2$$

$$\Rightarrow \quad h^2+k^2=r^2$$

Thus equation (1) becomes:

$$\Rightarrow \quad (x-h)^2+(y-k)^2=h^2+k^2$$

$$\Rightarrow \quad x^2+h^2-2hx+y^2+k^2-2ky=h^2+k^2$$

$$\Rightarrow \quad x^2+y^2-2hx-2ky=0$$

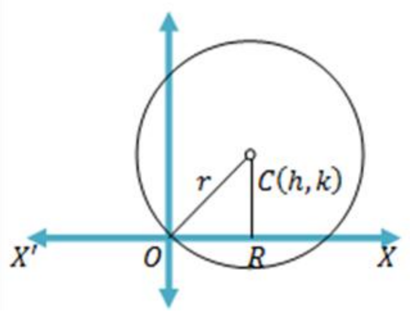


Fig. 5.3

(iii) When the circle passes through the origin and centre lies on the X-axis (see Fig. 5.4) i.e. $k=0$:

In this case radius $r=|h|$

Thus equation (1) becomes:

$$\Rightarrow \quad (x-h)^2+(y-0)^2=h^2$$

$$\Rightarrow \quad x^2+h^2-2hx+y^2=h^2$$

$$\Rightarrow \quad x^2+y^2-2hx=0$$

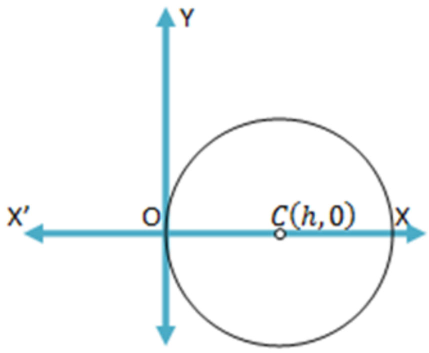


Fig. 5.4

(iv) When the circle passes through the origin and centre lies on the Y-axis(see Fig. 5.5) i.e. $h = 0$:

In this case radius $r = |k|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-0)^2 + (y-k)^2 = k^2$

$\Rightarrow \quad x^2 + y^2 + k^2 - 2ky = k^2$

$\Rightarrow \quad x^2 + y^2 - 2ky = 0$

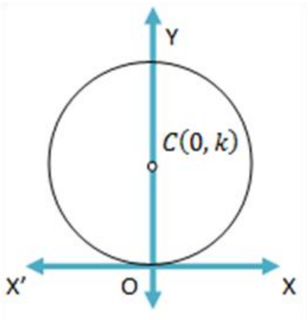


Fig. 5.5

(v) When the circle touches the X-axis (see Fig. 5.6):
In this case radius $r = |k|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2 + (y-k)^2 = k^2$

$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + k^2 - 2ky = k^2$

$\Rightarrow \quad x^2 + y^2 - 2hx - 2ky + h^2 = 0$

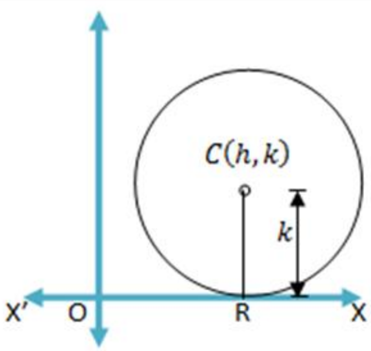


Fig. 5.6

(vi) When the circle touches the Y-axis (see Fig. 5.7):
In this case radius $r = |h|$
Thus equation (1) becomes:

$$\Rightarrow \quad (x-h)^2 + (y-k)^2 = h^2$$
$$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + k^2 - 2ky = h^2$$
$$\Rightarrow \quad x^2 + y^2 - 2hx - 2ky + k^2 = 0$$

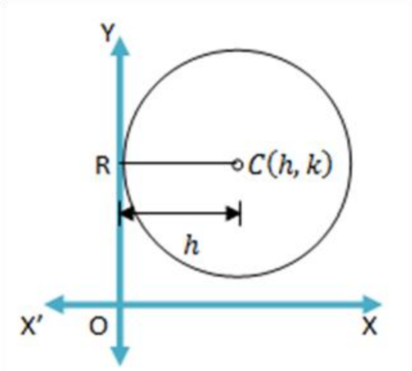


Fig. 5.7

(vii) When the circle touches both the axes (see Fig. 5.8):
In this case radius $r = |h| = |k|$
Thus equation (1) becomes:

$$\Rightarrow \quad (x-h)^2 + (y-h)^2 = h^2$$
$$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + h^2 - 2hy = h^2$$
$$\Rightarrow \quad x^2 + y^2 - 2hx - 2hy + h^2 = 0$$

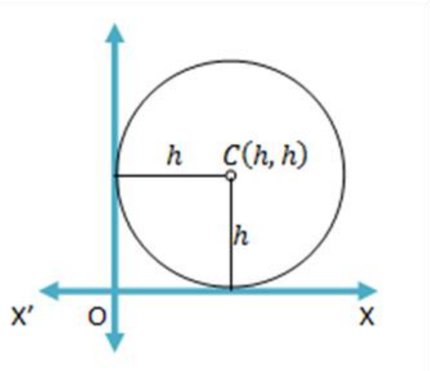


Fig. 5.8

General Equation of Circle: An equation of the form $x^2 + y^2 + 2gx + 2fy + c = 0$ is known as general equation of circle, where g , f and c are arbitrary constants.

To Convert General Equation into Standard Equation: Let the general equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (2)$$
$$\Rightarrow \quad (x^2 + 2gx) + (y^2 + 2fy) + c = 0$$
$$\Rightarrow \quad (x^2 + 2gx + g^2 - g^2) + (y^2 + 2fy + f^2 - f^2) + c = 0$$

$$\Rightarrow \quad (x+g)^2 - g^2 + (y+f)^2 - f^2 + c = 0$$
$$\Rightarrow \quad (x+g)^2 + (y+f)^2 = f^2 + g^2 - c$$
$$\Rightarrow \quad (x+g)^2 + (y+f)^2 = \left(\sqrt{g^2+f^2-c}\right)^2$$

which is the required standard form.

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

$h = -g$, $k = -f$ and $r = \sqrt{g^2+f^2-c}$.

Hence, centre of given circle (2) is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$.

We observe that the centre of circle (2) is $\left(-\frac{1}{2} \times Coefficient\ of\ x, -\frac{1}{2} \times Coefficient\ of\ y\right)$.

Example 1. Find the centre and radius of the following circles:

(i) $x^2 + y^2 + 2x + 4y - 4 = 0$ (ii) $x^2 + y^2 - 6x + 10y + 3 = 0$

(iii) $x^2 + y^2 - 3x - 5y - 1 = 0$ (iv) $2x^2 + 2y^2 + 5x - 6y + 2 = 0$

(v) $3x^2 + 3y^2 - 6x - 15y + 12 = 0$ (vi) $x^2 + y^2 - 12y + 6 = 0$

(vii) $x^2 + y^2 + 10x - 3 = 0$ (viii) $x^2 + y^2 + 7x - 9y = 0$

Sol.

(i) Given that equation of circle is

$x^2 + y^2 + 2x + 4y - 4 = 0$ (1)

Compare equation (1) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = 2$, $2f = 4$ and $c = -4$

i.e. $g = 1$, $f = 2$ and $c = -4$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (1) is $(-1, -2)$

and radius r of circle (1) is

$$r = \sqrt{1^2 + 2^2 - (-4)}$$
$$\Rightarrow \quad r = \sqrt{1+4+4}$$
$$\Rightarrow \quad r = \sqrt{9} = 3$$

(ii) Given that equation of circle is

$x^2 + y^2 - 6x + 10y + 3 = 0$ (2)

Compare equation (2) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -6$, $2f = 10$ and $c = 3$

i.e. $g = -3$, $f = 5$ and $c = 3$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (2) is $(3, -5)$

and radius r of circle (2) is

$$r=\sqrt{(-3)^2+5^2-3}$$
$$\Rightarrow \quad r=\sqrt{9+25-3}$$
$$\Rightarrow \quad r=\sqrt{31}$$

(iii) Given that equation of circle is

$$x^2+y^2-3x-5y-1=0 \qquad (3)$$

Compare equation (3) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=-3$, $2f=-5$ and $c=-1$

i.e. $g=-\frac{3}{2}$, $f=-\frac{5}{2}$ and $c=-1$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (3) is $\left(\frac{3}{2},\frac{5}{2}\right)$

and radius r of circle (3) is

$$r=\sqrt{\left(-\frac{3}{2}\right)^2+\left(-\frac{5}{2}\right)^2-(-1)}$$
$$\Rightarrow \quad r=\sqrt{\frac{9}{4}+\frac{25}{4}+1}$$
$$\Rightarrow \quad r=\sqrt{\frac{9+25+4}{4}}=\sqrt{\frac{38}{4}}$$
$$\Rightarrow \quad r=\sqrt{\frac{19}{2}}$$

(iv) Given that equation of circle is

$$2x^2+2y^2+5x-6y+2=0$$

Diving this equation by 2, we get

$$x^2+y^2+\frac{5}{2}x-3y+1=0 \qquad (4)$$

Compare equation (4) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=\frac{5}{2}$, $2f=-3$ and $c=1$

i.e. $g=\frac{5}{4}$, $f=-\frac{3}{2}$ and $c=1$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (4) is $\left(-\frac{5}{4},\frac{3}{2}\right)$

and radius r of circle (4) is

$$r=\sqrt{\left(\frac{5}{4}\right)^2+\left(-\frac{3}{2}\right)^2-1}$$

$\Rightarrow \quad r=\sqrt{\frac{25}{16}+\frac{9}{4}-1}$

$\Rightarrow \quad r=\sqrt{\frac{25+36-16}{16}}=\sqrt{\frac{45}{16}}$

$\Rightarrow \quad r=\frac{3\sqrt{5}}{4}$

(v) Given that equation of circle is

$3x^2+3y^2-6x-15y+12=0$

Diving this equation by 3, we get

$x^2+\ y^2-2x-5y+4=0$ (5)

Compare equation (5) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=-2$, $2f=-5$ and $c=4$

i.e. $g=-1$, $f=-\frac{5}{2}$ and $c=4$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$

Therefore, centre of circle (5) is $\left(1,\frac{5}{2}\right)$

and radius r of circle (5) is

$r=\sqrt{(-1)^2+\left(-\frac{5}{2}\right)^2-4}$

$\Rightarrow \quad r=\sqrt{1+\frac{25}{4}-4}$

$\Rightarrow \quad r=\sqrt{\frac{4+25-16}{4}}=\sqrt{\frac{13}{4}}$

$\Rightarrow \quad r=\frac{\sqrt{13}}{2}$

(vi) Given that equation of circle is

$x^2+y^2-12y+6=0$ (6)

Compare equation (6) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=0$, $2f=-12$ and $c=6$

i.e. $g=0$, $f=-6$ and $c=6$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (6) is $(0,6)$

and radius r of circle (6) is

$r=\sqrt{(0)^2+(-6)^2-6}$

$\Rightarrow \quad r=\sqrt{0+36-6}$

$\Rightarrow \quad r=\sqrt{30}$

(vii) Given that equation of circle is

$$x^2+y^2+10x-3=0 \qquad (7)$$

Compare equation (7) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=10$, $2f=0$ and $c=-3$

i.e. $g=5$, $f=0$ and $c=-3$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (7) is $(-5,0)$

and radius r of circle (7) is

$$r=\sqrt{(5)^2+(0)^2-(-3)}$$

$\Rightarrow \quad r=\sqrt{25+0+3}=\sqrt{28}$

$\Rightarrow \quad r=2\sqrt{7}$

(viii) Given that equation of circle is

$$x^2+y^2+7x-9y=0 \qquad (8)$$

Compare equation (8) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=7$, $2f=-9$ and $c=0$

i.e. $g=\frac{7}{2}$, $f=-\frac{9}{2}$ and $c=0$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (8) is $\left(-\frac{7}{2},\frac{9}{2}\right)$

and radius r of circle (8) is

$$r=\sqrt{\left(\frac{7}{2}\right)^2+\left(-\frac{9}{2}\right)^2-0}$$

$\Rightarrow \quad r=\sqrt{\frac{49}{4}+\frac{81}{4}}$

$\Rightarrow \quad r=\sqrt{\frac{49+81}{4}}=\sqrt{\frac{130}{4}}$

$\Rightarrow \quad r=\sqrt{\frac{65}{2}}$

Example 2. Find the equations of circles if their centres and radii are as follow:

(i) $(0,0)$, 2 (ii) $(2,0)$, 5 (iii) $(0,-3)$, 3

(iv) $(8,-4)$, 1 (v) $(3,6)$, 6 (vi) $(-2,-5)$, 10

Sol.

(i) Given that centre of circle is $(0,0)$ and radius is 2 i.e. $h=0$, $k=0$ and $r=2$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-0)^2+(y-0)^2=2^2$$
$$\Rightarrow \quad x^2+y^2=4$$
$$\Rightarrow \quad x^2+y^2-4=0$$

which is the required equation of circle.

(ii) Given that centre of circle is $(2,0)$ and radius is 5 i.e. $h=2$, $k=0$ and $r=5$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-2)^2+(y-0)^2=5^2$$
$$\Rightarrow \quad x^2+4-4x+y^2=25$$
$$\Rightarrow \quad x^2+y^2-4x-21=0$$

which is the required equation of circle.

(iii) Given that centre of circle is $(0,-3)$ and radius is 3 i.e. $h=0$, $k=-3$ and $r=3$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-0)^2+(y-(-3))^2=3^2$$
$$\Rightarrow \quad x^2+(y+3)^2=3^2$$
$$\Rightarrow \quad x^2+y^2+9+6y=9$$
$$\Rightarrow \quad x^2+y^2+6y=0$$

which is the required equation of circle.

(iv) Given that centre of circle is $(8,-4)$ and radius is 1 i.e. $h=8$, $k=-4$ and $r=1$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-8)^2+(y-(-4))^2=1^2$$
$$\Rightarrow \quad (x-8)^2+(y+4)^2=1^2$$
$$\Rightarrow \quad x^2+64-16x+y^2+16+8y=1$$
$$\Rightarrow \quad x^2+y^2-16x+8y+79=0$$

which is the required equation of circle.

(v) Given that centre of circle is $(3,6)$ and radius is 6 i.e. $h=3$, $k=6$ and $r=6$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-3)^2+(y-6)^2=6^2$$
$$\Rightarrow \quad x^2+9-6x+y^2+36-12y=36$$
$$\Rightarrow \quad x^2+y^2-6x-12y+9=0$$

which is the required equation of circle.

(vi) Given that centre of circle is $(-2,-5)$ and radius is 10 i.e. $h=-2$, $k=-5$ and $r=10$.

We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$

$\Rightarrow \quad (x-(-2))^2+(y-(-5))^2=10^2$

$\Rightarrow \quad (x+2)^2+(y+5)^2=100$

$\Rightarrow \quad x^2+4+4x+y^2+25+10y=100$

$\Rightarrow \quad x^2+y^2+4x+10y-71=0$

which is the required equation of circle.

<u>Process to find the equation of circle which passes through the three given points</u>:

Suppose that the circle passes through the following three points $(x_1\,,y_1)$, $(x_2\,,y_2)$ and $(x_3\,,y_3)$.

Step 1: Write the general equation of the circle, i.e.,

$$x^2+y^2+2gx+2fy+c=0 \qquad (1)$$

Step 2: Since the circle passes through the points $(x_1\,,y_1)$, $(x_2\,,y_2)$ and $(x_3\,,y_3)$, so these points will satisfy the equation (1). Hence substitute x-coordinates and y-coordinates of these points one by one in equation (1) and we will get three linear equations in three unknowns. The three linear equations are as follow

$$x_1{}^2+y_1{}^2+2gx_1+2fy_1+c=0 \qquad (2)$$

$$x_2{}^2+y_2{}^2+2gx_2+2fy_2+c=0 \qquad (3)$$

$$x_3{}^2+y_3{}^2+2gx_3+2fy_3+c=0 \qquad (4)$$

Step 3: Solve equations (2), (3) and (4) for g, f and c.

Step 4: Use the values of g, f and c in equation (1), we will get the required equation of the circle.

Example 3. Find the equations of circles which passes through the following three points:

(i) $(0\,,0)$, $(3\,,0)$ and $(0\,,4)$

(ii) $(0\,,0)$, $(-5\,,2)$ and $(3\,,6)$

(iii) $(2\,,-3)$, $(1\,,4)$ and $(-1\,,2)$

Sol. (i) Let the equation of the circle is

$$x^2+y^2+2gx+2fy+c=0 \qquad (1)$$

Circle (1) passes through the point $(0\,,0)$, therefore

$$0^2+0^2+2g(0)+2f(0)+c=0$$

i.e. $\quad c=0 \qquad (2)$

Circle (1) passes through the point $(3\,,0)$, therefore

$$3^2+0^2+2g(3)+2f(0)+0=0$$

$\Rightarrow \quad 9+6g=0$

$\Rightarrow \qquad g = -\frac{9}{6}$

$\Rightarrow \qquad g = -\frac{3}{2}$ (3)

Also, the circle (1) passes through the point $(0\,,4)$, therefore

$$0^2 + 4^2 + 2g(0) + 2f(4) + 0 = 0$$

$\Rightarrow \qquad 16 + 8f = 0$

$\Rightarrow \qquad f = -\frac{16}{8}$

$\Rightarrow \qquad f = -2$ (4)

Using (2), (3) and (4) in equation (1), we have

$$x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2(-2)y + 0 = 0$$

$\Rightarrow \qquad x^2 + y^2 - 3x - 4y = 0$

which is the required equation of the circle.

(ii) Let the equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Circle (1) passes through the point $(0\,,0)$, therefore

$$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$$

i.e. $\quad c = 0$ (2)

Circle (1) passes through the point $(-5\,,2)$, therefore

$$(-5)^2 + 2^2 + 2g(-5) + 2f(2) + c = 0$$

$\Rightarrow \qquad 25 + 4 - 10g + 4f + 0 = 0$

$\Rightarrow \qquad -10g + 4f = -29$ (3)

Also, the circle (1) passes through the point $(3\,,6)$, therefore

$$3^2 + 6^2 + 2g(3) + 2f(6) + c = 0$$

$\Rightarrow \qquad 9 + 36 + 6g + 12f + 0 = 0$

$\Rightarrow \qquad 6g + 12f = -45$

$\Rightarrow \qquad 2g + 4f = -15$ (4)

Solving equations (3) and (4):

Applying $equation\ (3) + 5 \times equation\ (4)$, we get

$$\begin{aligned} -10g + 4f &= -29 \\ \underline{10g + 20f} &\underline{= -75} \\ 24f &= -104 \end{aligned}$$

$\Rightarrow \qquad f = -\frac{104}{24}$

$\Rightarrow \qquad f = -\frac{13}{3}$ (5)

Using equation (5) in equation (4), we get

$$2g + 4\left(-\frac{13}{3}\right) = -15$$

$$\Rightarrow \quad 6g - 52 = -45$$

$$\Rightarrow \quad g = \frac{7}{6} \qquad (6)$$

Now, using equations (2), (5) and (6) in equation (1), we have

$$x^2 + y^2 + 2\left(\frac{7}{6}\right)x + 2\left(-\frac{13}{3}\right)y + 0 = 0$$

$$\Rightarrow \quad x^2 + y^2 + \frac{7}{3}x - \frac{26}{3}y = 0$$

$$\Rightarrow \quad 3x^2 + 3y^2 + 7x - 26y = 0$$

which is the required equation of the circle.

(iii) Let the equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Circle (1) passes through the point $(2\,, -3)$, therefore

$$2^2 + (-3)^2 + 2g(2) + 2f(-3) + c = 0$$

$$\Rightarrow \quad 4 + 9 + 4g - 6f + c = 0$$

$$\Rightarrow \quad 4g - 6f + c = -13 \qquad (2)$$

Circle (1) passes through the point $(1\,, 4)$, therefore

$$1^2 + 4^2 + 2g(1) + 2f(4) + c = 0$$

$$\Rightarrow \quad 1 + 16 + 2g + 8f + c = 0$$

$$\Rightarrow \quad 2g + 8f + c = -17 \qquad (3)$$

Also, the circle (1) passes through the point $(-1\,, 2)$, therefore

$$(-1)^2 + 2^2 + 2g(-1) + 2f(2) + c = 0$$

$$\Rightarrow \quad 1 + 4 - 2g + 4f + c = 0$$

$$\Rightarrow \quad -2g + 4f + c = -5 \qquad (4)$$

Applying $equation\ (2) - equation\ (3)$, we get

$$2g - 14f = 4 \qquad (5)$$

Applying $equation\ (3) - equation\ (4)$, we get

$$4g + 4f = -12 \qquad (6)$$

Applying $2 \times equation\ (5) - equation\ (6)$, we get

$$-28f - 4f = 8 + 12$$

$$\Rightarrow \quad -32f = 20$$

$$\Rightarrow \quad f = -\frac{5}{8} \qquad (7)$$

Using equation (7) in equation (5), we get

$$2g - 14\left(-\frac{5}{8}\right) = 4$$

$$\Rightarrow \quad 2g = -\frac{35}{4} + 4$$

$$\Rightarrow \quad g = -\frac{19}{8} \qquad (8)$$

Using equations (7) and (8) in equation (4), we get

$$-2\left(-\frac{19}{8}\right)+4\left(-\frac{5}{8}\right)+c=-5$$

$\Rightarrow$ $$\frac{19}{4}-\frac{5}{2}+c=-5$$

$\Rightarrow$ $$c=-5-\frac{19}{4}+\frac{5}{2}$$

$\Rightarrow$ $$c=-\frac{29}{4} \qquad (9)$$

Now, using equations (7), (8) and (9) in equation (1), we have

$$x^2+y^2+2\left(-\frac{19}{8}\right)x+2\left(-\frac{5}{8}\right)y-\frac{29}{4}=0$$

$\Rightarrow$ $$4x^2+4y^2-19x-5y-29=0$$

which is the required equation of the circle.

<u>Equation of Circle in Diametric Form</u>: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the end points of diameter of a circle then equation of circle is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Example 4. Find the equations of circles if end points of their diameters are as follow:

(i) $(1,5)$ and $(3,6)$ (ii) $(1,0)$ and $(-2,-5)$

(iii) $(0,0)$ and $(8,-6)$ (iv) $(-3,2)$ and $(-7,9)$

Sol.

(i) Given that end points of diameter of circle are $(1,5)$ and $(3,6)$.
Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get
$x_1=1$, $y_1=5$, $x_2=3$ and $y_2=6$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\Rightarrow$ $$(x-1)(x-3)+(y-5)(y-6)=0$$

$\Rightarrow$ $$x^2-3x-x+3+y^2-6y-5y+30=0$$

$\Rightarrow$ $$x^2+y^2-4x-11y+33=0$$

which is the required equation of circle.

(ii) Given that end points of diameter of circle are $(1,0)$ and $(-2,-5)$.
Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get
$x_1=1$, $y_1=0$, $x_2=-2$ and $y_2=-5$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\Rightarrow$ $$(x-1)(x-(-2))+(y-0)(y-(-5))=0$$

$\Rightarrow$ $$(x-1)(x+2)+y(y+5)=0$$

$\Rightarrow$ $$x^2+2x-x-2+y^2+5y=0$$

$\Rightarrow$ $$x^2+y^2+x+5y-2=0$$

which is the required equation of circle.

(iii) Given that end points of diameter of circle are $(0,0)$ and $(8,-6)$.

Comparing these points with (x_1, y_1) and $(x_2\ , y_2)$ respectively, we get
$x_1 = 0$, $y_1 = 0$, $x_2 = 8$ and $y_2 = -6$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
$$\Rightarrow \quad (x-0)(x-8)+(y-0)(y-(-6))=0$$
$$\Rightarrow \quad x(x-8)+y(y+6)=0$$
$$\Rightarrow \quad x^2-8x+y^2+6y=0$$
$$\Rightarrow \quad x^2+y^2-8x+6y=0$$

which is the required equation of circle.

(iv) Given that end points of diameter of circle are $(-3,2)$ and $(-7,9)$.
Comparing these points with (x_1, y_1) and $(x_2\ , y_2)$ respectively, we get
$x_1 = -3$, $y_1 = 2$, $x_2 = -7$ and $y_2 = 9$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
$$\Rightarrow \quad (x-(-3))(x-(-7))+(y-2)(y-9)=0$$
$$\Rightarrow \quad (x+3)(x+7)+(y-2)(y-9)=0$$
$$\Rightarrow \quad x^2+7x+3x+21+y^2-9y-2y+18=0$$
$$\Rightarrow \quad x^2+y^2+10x-11y+39=0$$

which is the required equation of circle.

EXERCISE-I

1. Equation of circle with centre at $(2,0)$ and radius 7 is:
   (a) $x^2 + 4 - 4x + y^2 = 14$ (b) $x^2 + 4 - 4x + y^2 = 49$
   (c) $x^2 - 4 + 4x + y^2 = 49$ (d) None of these
2. Equation of circle whose centre is origin and radius v is:
   (a) $x^2 + y^2 + 2gx + 2fy + c = 0$ (b) $x^2 + y^2 = v^2$
   (c) $x^2 - 2vx + y^2 = v^2$ (d) $x^2 + y^2 = 0$
3. Equation of circle in diametric form is:
   (a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ (b) $(x - x_1)(y - y_1) + (x - x_2)(y - y_2) = 0$
   (c) $(x - y_1)(x - x_2) + (y - x_1)(y - x_2) = 0$ (d) $(x - y)(x - y) + (y_1 - x_1)(y_2 - x_2) = 0$
4. Find the centres and radii of the following circles:
   (i) $9x^2 + 9y^2 - 12x - 30y + 24 = 0$
   (ii) $x^2 + y^2 - 6y - 24 = 0$
   (iii) $x^2 + y^2 + 20x - 5 = 0$
5. Find the equations of circles whose centres and radii are as follow:
   (i) $(8,8)$, 2 (ii) $(6,3)$, 6
6. Find the equations of circles if end points of their diameters are as follows:
   (i) $(2,6)$ and $(3,16)$ (ii) $(-1,-1)$ and $(4,5)$
7. Find the equations of circles which passes through the following three points:

(i) $(-3, 0)$, $(0, 5)$ and $(0, 0)$
(ii) $(2, 1)$, $(0, 5)$ and $(-1, 2)$
(iii) $(-6, 5)$, $(-3, -4)$ and $(2, 1)$

ANSWERS

1. (b) 2. (b)
3. (a)
4. (i) $\left(\frac{2}{3}, \frac{5}{3}\right); \sqrt{\frac{5}{9}}$ (ii) $(0,3); \sqrt{33}$
   (iii) $(-10,0); \sqrt{105}$
5. (i) $x^2 + y^2 - 16x - 16y + 124 = 0$
   (ii) $x^2 + y^2 - 12x - 6y + 9 = 0$
6. (i) $x^2 + y^2 - 5x - 22y + 102 = 0$
   (ii) $x^2 + y^2 - 3x - 4y - 9 = 0$
7. (i) $x^2 + y^2 + 3x - 5y = 0$
   (ii) $x^2 + y^2 - 2x - 6y + 5 = 0$
   (iii) $x^2 + y^2 + 6x - 2y - 15 = 0$

Software

5.2 MATLAB Or SciLab software

Introduction to MATLAB

In the introduction we will describe how MATLAB handles numerical expressions and mathematical formulae. The name MATLAB stands for MATrix LABoratory. Primarily, it is developed by Cleve Moler in the 1970's. MATLAB was written originally to provide easy access to matrix software and derived from FORTRAN subroutines LINPACK (linear system package) and EISPACK (Eigen system package). It is again rewritten in C in the 1980's with more functionality, which included plotting routines. Then the MathWorks Inc. was created (1984) to market to continue development of MATLAB. It has been designed to supersede LINPACK and EISPACK.

MATLAB is a high-performance language for technical computing. It provides an interactive environment to perform reports and data analysis. It also allows the implementation of computing algorithms, plotting graphs and other matrix functions. It contains built-in editing and debugging tools. These are the excellent features of MATLAB for teaching and research.

MATLAB has many advantages compared to conventional computer languages (e.g., C, FORTRAN) for solving technical problems. MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. It has powerful built-in routines that enable a very wide variety of computations. It also has easy to use graphics commands that make the visualization of results immediately available. There are toolboxes for signal processing, symbolic computation, control theory, simulation, optimization, and several other fields of applied science and engineering.

Starting with MATLAB

After completion of installation process and logging into your account, we can enter in MATLAB by double-clicking on the MATLAB shortcut *icon* available on our desktop. When we start MATLAB, a special window called the MATLAB desktop appears which contains some *other* windows. The major tools within the MATLAB desktop are:

1. **The Command Window**: It is used to enter commands and data.
2. **The Graphic Window**: It is used to display plots and graphs.
3. **The Edit Window**: It which is used to create and modify M-files. (M-files are files that contain a program or script in MATLAB commands).
4. **The Command History**: It displays a log of statement that is ran in the current and previous MATLAB sessions.
5. **The Workspace**: It contains variables that are created or imported by users into MATLAB from data files or other programs.
6. **The Current Directory**: It is a reference location that MATLAB uses to find files.
7. **The Help Browser**: It is a Web browser integrated into the MATLAB desktop that displays HTML documents.
8. **The Start button**: It provides easy access to tools, demos and documentation for MathWorks products. Through this users can create and run MATLAB shortcuts, which are groups of MATLAB statements.

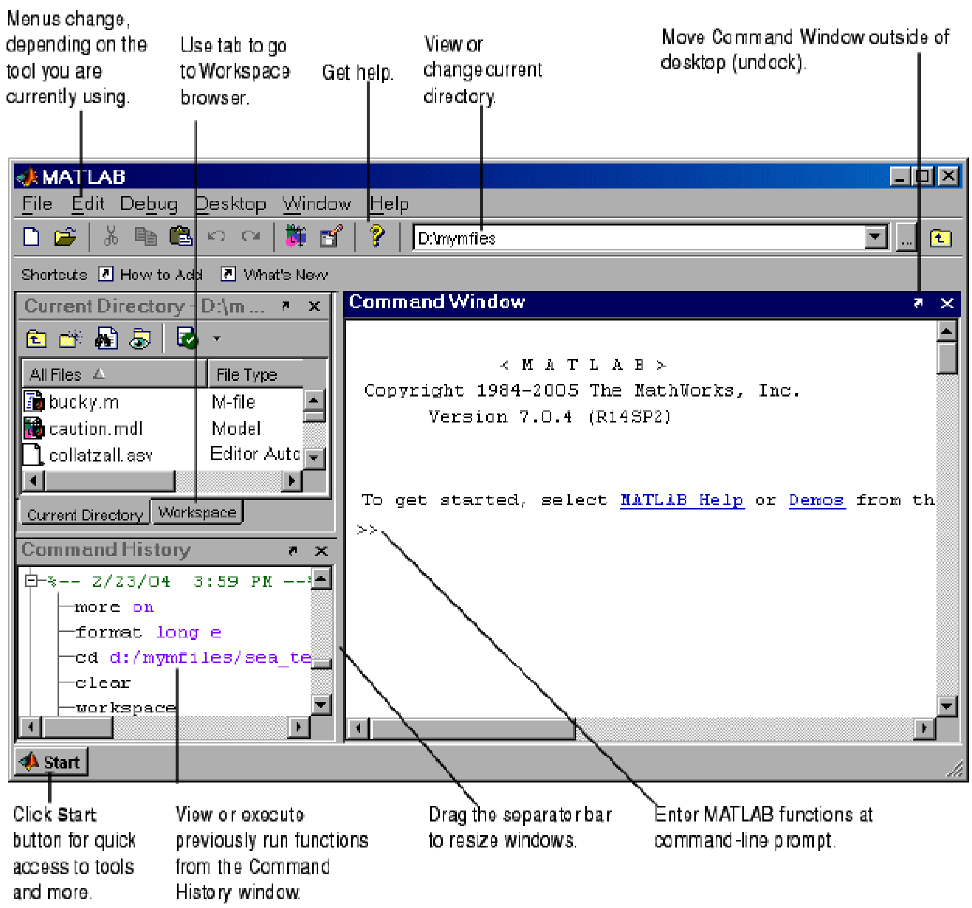


Fig. 5.9: The graphical interface to the MATLAB (Version 7.0.4) workspace

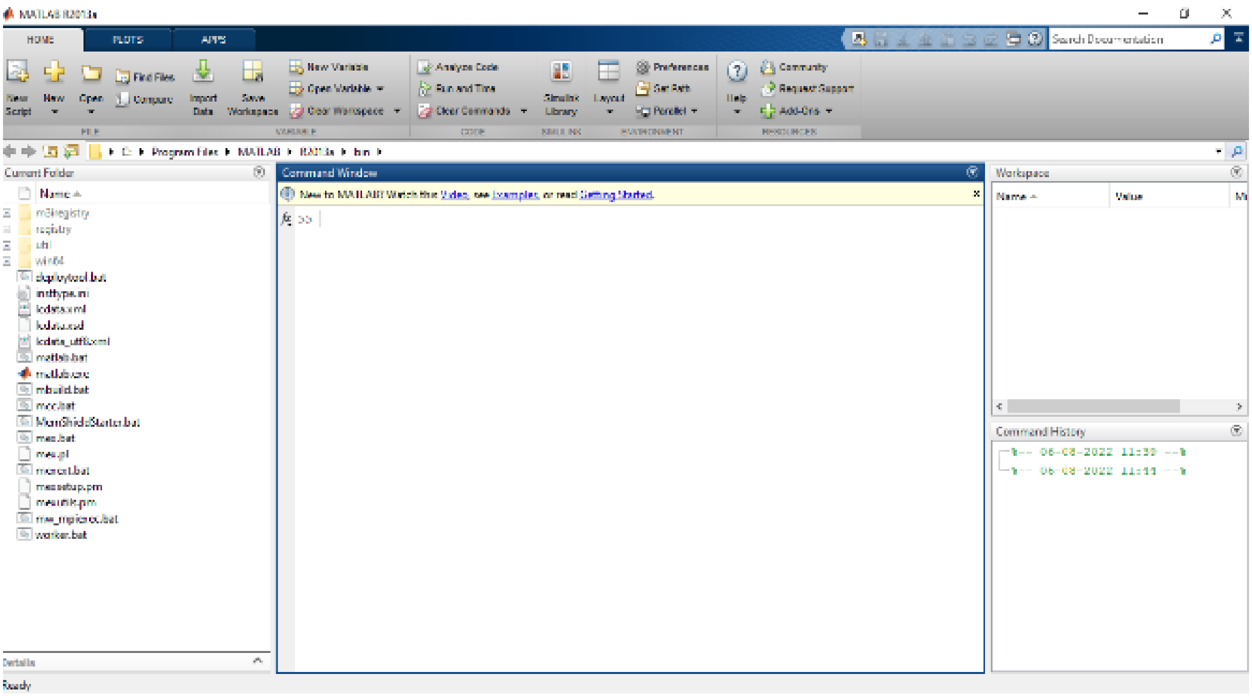


Fig. 5.10: The graphical interface to the MATLAB (Version R2013a) workspace

When MATLAB is started for the first time, the screen looks like the one that shown in the Fig. 5.9 or Fig. 5.10. This illustration also shows the default configuration of the MATLAB desktop. You can customize the arrangement of tools and documents to suit your needs. The screen will produce the MATLAB prompt **>>** (or **EDU >>**), which indicates that MATLAB is waiting for a command to be entered.

>> for full version

and

EDU>> for educational version.

Quitting MATLAB

In order to quit MATLAB, type **quit** or **exit** after the prompt, followed by pressing the enter or return key.

Entering Commands

To execute commands, every command has to be followed by enter key. MATLAB commands are case sensitive and **lower case** letters are used throughout. To execute an M-file (such as Demo_1.m), simply enter the name of the file without its extension (as in Demo_1).

The Semicolon Symbol (;)

If the semicolon symbol (;) is typed at the end of a command, the output of the command is not displayed.

The Percent Symbol (%)

If the percent symbol (%) is typed at the beginning of a line, then the line is designated as a comment. When the enter key is pressed, the line is not executed in this case.

The clc Command

Typing **clc** command and pressing enter key cleans the command window. Once the clc command is executed, a clear window is displayed.

The clear Command

The **clear** command remove all the variables from the memory.

Help

To obtain help on a particular topic in the MATLAB-list of built-in functions, e.g., Determinant, type help det after prompt.

Special Variable Names and Constants

1. **ans** It represents a value computed by an expression but not stored in a variable name.
2. **i, j** Imaginary unit/operator defined as $\sqrt{-1}$.
3. **inf** Infinity (∞)
4. **eps** Smallest floating point number.
5. **pi** π = 3.141592653589793
6. **NaN** Stands for not a number. E.g., 0/0.
7. **clock** It represents the current time in a row vector of six elements containing year, month, day, hour, minute, and seconds.
8. **date** It represents the current date in a character string format.

Note: (i) Overwriting/using these variables and constants should be avoided in programming.

(ii) MATLAB is a case sensitive language for function, script and variable names for all the platforms. For instance, Ab, ab, aB and AB are the names of four different variables.

Arithmetic Operations

Name of Arithmetic Operation	**Symbol**	**Exapmle**
Addition	+	10+5 = 15
Subtraction	-	10-5 = 5
Multiplication	*	10*5 = 50
Right Division	/	10/5 = 2
Left Division	\	10\5 = 5/10 = ½
Exponentiation	^	10^5 = 10^5 = 100000

Display Formats

Command	**Description**
format short	Fixed point with four decimal digits
format short e	Scientific notation with four decimal digits
format short g	Best of five digits fixed or floating point
format long	Fixed point with fourteen decimal digits
format long e	Scientific notation with fifteen decimal digits
format long g	Best of fifteen digits fixed or floating point
format bank	Two decimal digits
format compact	Suppresses the display of blank lines
format loose	Keeps the display of blank lines (default)

Trigonometric and Inverse Trigonometric Functions

Function Name	**Description**
sin(x)	Sine of argument x in radians
cos(x)	Cosine of argument x in radians
tan(x)	Tangent of argument x in radians
sec(x)	Secant of argument x in radians
csc(x)	Cosecant of argument x in radians
cot(x)	Cotangent of argument x in radians
asin(x)	Inverse sine, results in radians.
acos(x)	Inverse cosine, results in radians.
atan(x)	Inverse tangent, results in radians.
asec(x)	Inverse secant, results in radians.
acsc(x)	Inverse cosecant, results in radians.
acot(x)	Inverse cotangent, results in radians.

Some General Commands

Command Name	**Description**
Clc	It clears the command window.
Clear	It clears the Workspace, all variables are removed.
clear all	Same as the command clear.
clear a b c	It clears only the variables a, b and c from the Workspace.
Clf	It clears the figure window.
Who	Lists variables currently in the Workspace.
whos	Lists variables currently in the Workspace with their sizes.

Using MATLAB as a calculator

Let's start with a very simple interactive calculation at the very beginning. For example, suppose that we have to calculate an expression, $2 \times 9 - 5$, then we will type it at the prompt command (>>) as follows:

```
>> 2*9-5
ans =
      13
```

Here we have not assigned any specify variable name as an output variable, so MATLAB used a default variable **ans** (short for answer) to store the results of the current expression. The variable **ans** is created (or overwritten, if it is already existed). To avoid this, we may assign a value to a variable name. For example, if we type

```
>> a = 2*9-5
a =
   13
```

Here the value of the expression is assigned to the variable a. This variable name can be used to refer to the results of the previous computations. Therefore, computing 2a+10 will result in

```
>> 2*a+10
ans =
      36
```

Note: From above we can learn how to create a variable in MATLAB. So the syntax to create a variable in MATLAB is:

variable name = a value or an expression

Further, to evaluate the value of $\sin\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{\pi}{2}\right)$ and to assign the value of this expression into the variable p, we will type the following:

```
>> p = sin(pi/3)+2*cos(pi/2)
p =
    0.8660
```

<u>Merits</u>

1. MATLAB is relatively easy to learn.
2. MATLAB may behave as a *calculator* or as a *programming language.*
3. A large set of toolboxes is available. A toolbox is a collection of MATLAB functions specific for a subject, e.g. the signal processing toolbox or the control toolbox. Over 8000 functions available for various disciplines.
4. At any time, variables (results of simulations) are stored in the workspace for debugging and inspection.
5. MATLAB combine nicely calculation and it has excellent visualization (plots) capabilities.
6. MATLAB can solve complex algebraic equations.
7. MATLAB can process and communicate the signals.
8. MATLAB is interpreted (not compiled), errors are easy to fix.
9. MATLAB is optimized to be relatively fast when performing matrix operations.
10. Quick code development.

Demerits

1. Very expensive for non students, although there are some free clones, such as Octave or Scilab that are MATLAB compatible (but not 100%).
2. MATLAB is not a *general* purpose programming language such as C, C++, or FORTRAN.
3. MATLAB is designed for scientific computing, and is not well suitable for other applications.
4. MATLAB is an interpreted language, slower than a compiled language such as C++.
5. Code execution can be slow if programmed carelessly without vectorization.
6. MATLAB commands are specific for MATLAB usage. Most of them do not have a direct equivalent with other programming language commands.

<u>Introduction to Scilab</u>:

Scilab is a scientific software package developed by INRIA and ENPC. It is an open-source software that is used for data analysis and computation. It is also an alternative for MATLAB as this is not open-source. Scilab is named as Scientific Laboratory which resolves the problem related to numeric data and scientific visualization. It is capable of interactive calculations as well as automation of computations through programming. It provides all basic operations on matrices through built-in functions so that the trouble of developing and testing code for basic operations are completely avoided. Further, the numerous toolboxes that are available for various specialized applications make it an important tool for research. Being compatible with Matlab, all available Matlab M-files can be directly used in Scilab with the help of the Matlab to Scilab translator. The greatest features of Scilab are that it is multiplatform and is free. It is available for many operating systems including Windows, Linux and MacOS X. Some basic features of Scilab are given below:

1. It is capable to solve different algebraic equations.
2. It supports the development of certain complicated algorithms.
3. Capable of the model the previous computations.
4. Performs visualization in Bar Graphs, lines, Histograms, MathML annotation.

When we start up Scilab, we see a window shown in Fig. 5.11.

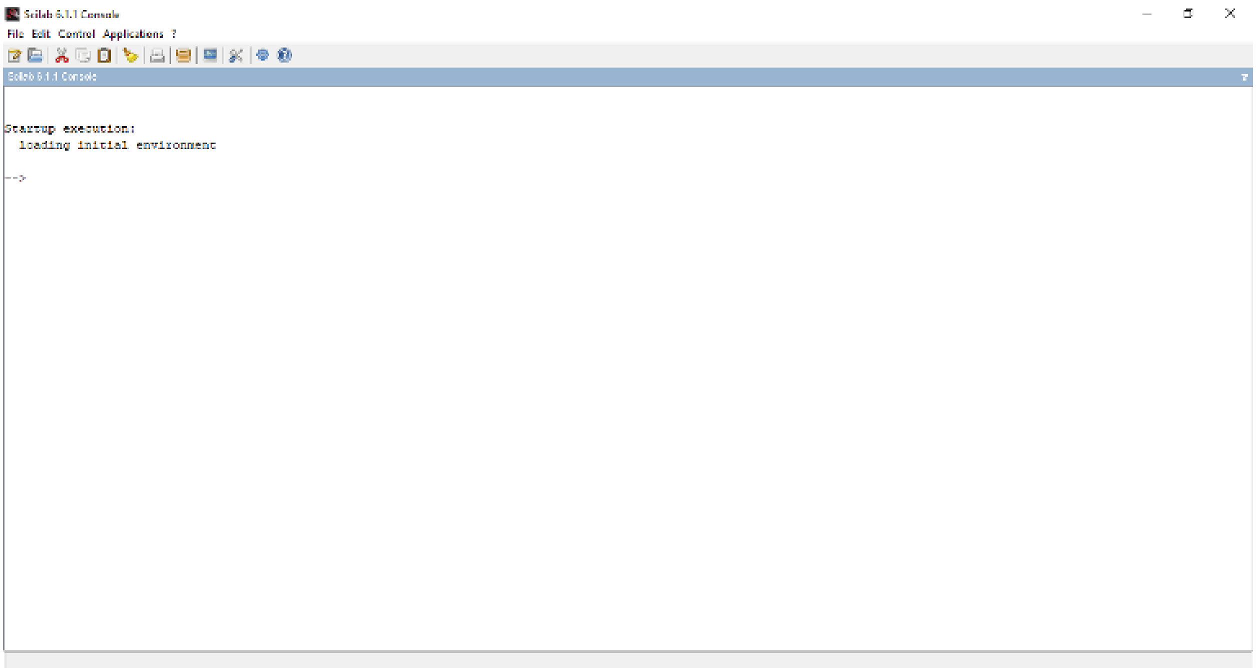


Fig. 5.11

The user enters Scilab commands after the prompt **-->**. But many of the commands are also available through the menu at the top. The most important menu for a beginner is the “Help” menu. Clicking on the “Help” menu opens up the *Help Browser*, showing a list of topics on which help is available (see the Fig. 5.12).

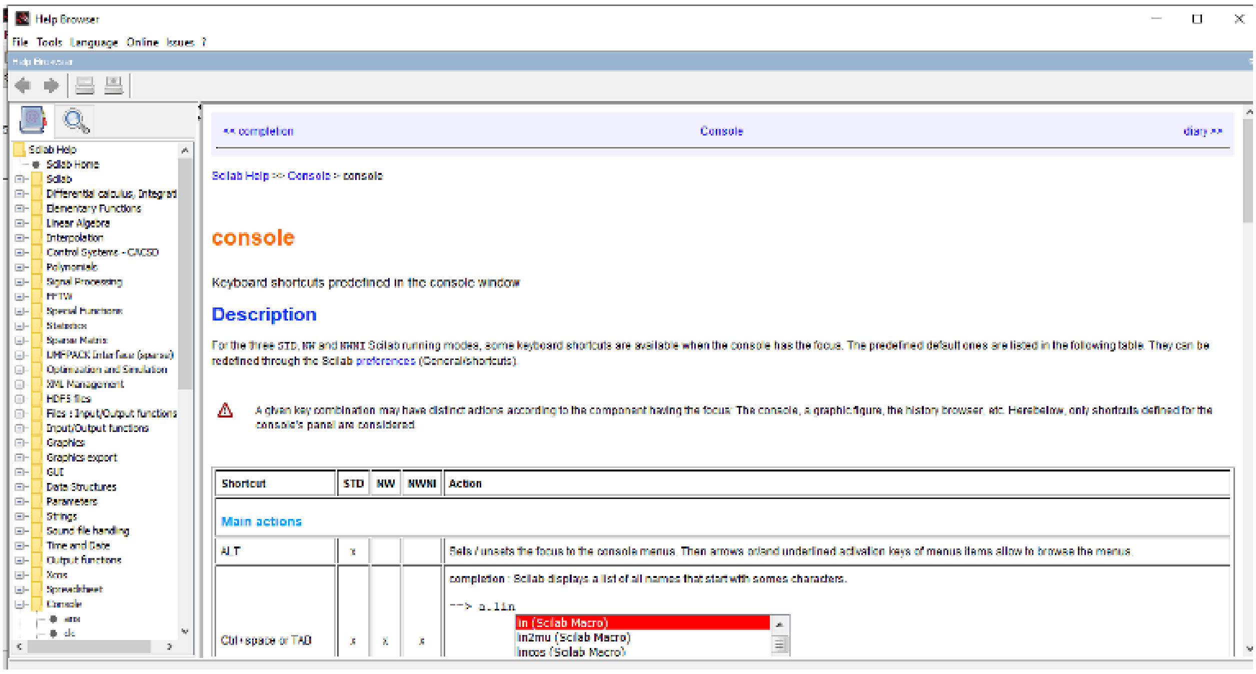


Fig. 5.12

Clicking on the relevant topic takes you to hyperlinked documents similar to web pages. The Help Browser has two tabs – *Table of Contents* and *Search*.

Table of Contents contains an alphabetically arranged list of topics and we may use the Search tab to search the help for particular topic by typing the topic in it.

Some calculations in Scilab and their results are given below:

```
--> 2+3
 ans  =

   5.

--> a=2+3
 a  =

   5.

--> p=sin(%pi/3)+2*cos(%pi/3)
 p  =

   1.8660254

--> b=2+3-5*6
 b  =

  -25.
```

Here we can see that **ans** is used as the default variable. Further, the predefined constants in Scilab are shown in the table given below:

Constant	Meaning
%pi	$\pi = 3.14159 \ldots$
%e	$e = 2.71828 \ldots$
%i	iota i.e. $\sqrt{-1}$
%eps	Epsilon
%inf	Infinity i.e. ∞
%nan	Not a number

Trigonometric and Inverse Trigonometric Functions

Function Name	**Description**
sin(x)	Sine of argument x in radians
cos(x)	Cosine of argument x in radians
tan(x)	Tangent of argument x in radians
sec(x)	Secant of argument x in radians
csc(x)	Cosecant of argument x in radians
cotg(x)	Cotangent of argument x in radians
asin(x)	sine inverse (radians)
acos(x)	element wise cosine inverse (radians)
atan(x)	2^{nd} quadrant and 4^{th} quadrant inverse tangent
asec(x)	computes the element-wise inverse secant of the argument
acsc(x)	computes the element-wise inverse cosecant of the argument
acot(x)	computes the element-wise inverse cotangent of the argument

EXERCISE-II

1. How to write infinity in MATLAB?
   (a) inf (b) infinity
   (c) undefined (d) None of these
2. How to write imaginary unit *iota* in Scilab?
   (a) i (b) j

(c) $\%i$ (d) None of these

3. What will be the output of 0/0 is MATLAB?

(a) inf (b) eps

(c) NaN (d) None of these

4. Is MATLAB case sensitive language?
5. How to assign the value of expression 2+9×3-5 to the variable q in MATLAB?
6. How to assign the value of expression 5×3-8×4 to the variable p in Scilab?
7. Write the syntaxes of tangent and inverse tangent function into MATLAB.
8. Write the syntaxes of cotangent and inverse cotangent function into Scilab.

ANSWERS

1. (a) 2. (c)
3. (c) 4. Yes
5. $q = 2 + 9 * 3 - 5$ 6. $p = 5 * 3 - 8 * 4$
7. tan $(value)$ and $a$tan $(value)$
8. $cotg(x)$ and $acot(x)$

Question Bank

Q.1. i^4 is equal to ______.
(a) -1
(b) 1
(c) i
(d) $-i$

Ans:- (b)

Q.2. i^{25} is equal to ______.
(a) i
(b) $-i$
(c) -1
(d) 1

Ans:- (a)

Q.3. If z= $x + iy$, then the conjugate of z is ______.
(a) $x - iy$
(b) $x + iy$
(c) $x^2 + y^2$
(d) None of these

Ans:- (a)

Q.4. If $z = 1 + i$, then $|z| =$ _____.
(a) 2
(b) $\sqrt{2}$
(c) 4
(d) -2

Ans:- (b)

Q.5. The argument of $1 + i$ is ______.
(a) 90^0
(b) 45^0
(c) 30^0
(d) 60^0

Ans:- (b)

Q.6. The complex number $2 - 3i$ lies in ______ quadrant.
(a) I
(b) II
(c) III
(d) IV

Ans:- (d)

Q.7. The complex number $3i$ lies on ______.
(a) X-axis
(b) Y-axis
(c) Both axis
(d) None of these

Ans:- (b)

Q.8. If $2x + (3x + y)i = 4 + 10i$, the value of (x, y) is ______.
(a) (2,4)
(b) (4,2)
(c) (-4,2)
(d) (-2,4)

Ans:- (a)

Q.9. The rectangular form of $4(Cos300^0 + iSin300^0)$ is ______.
(a) $2\text{-}2\sqrt{3}$ i
(b) $2+2\sqrt{3}$ i
(c) $-2+2\sqrt{3}$ i
(d) None of these

Ans:- (a)

Q.10. If $z = 1 + \sqrt{3}\, i$, then $|z|$ is ______.
(a) 60^0
(b) 30^0
(c) 45^0
(d) 90^0

Ans:- (a)

Q.11. If z_1= 2+ 3i and z_2 = 3 +2i , then z_1 +z_2 is equal to ______.
(a) 5+5i
(b) 5 -5i
(c) -5 +5i
(d) None of these

Ans:- (a)

Q.12. If z_1= 3-4i , z_2 = 2+5i then z_1z_2 is ______.
(a) 26 +7i
(b) 26 -7i
(c) -26 -7i
(d) None of these

Ans:- (a)

Q.13. The value of x and y is ______ , if $2 + (x + iy) = 5 - i$.
(a) (0,5)
(b) (5,0)
(c) (-5,0)
(d) None of these

Ans:- (b)

Q.14. Find the argument of $1 - i$ is ______.
(a) $\frac{\pi}{2}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{3}$

Ans:- (b)

Q.15. If $z_1 = 1 - i \; and \; z_2 = -2 + 4i$, then $Re\left(\frac{z_1 \, z_2}{\overline{z_1}}\right)$ is ______.

(a) -4
(b) 4
(c) 2
(d) -2

Ans:- (c)

Q.16. The modulus of $\frac{3+i}{2-i}$ is ______.

(a) 2
(b) -2
(c) 3
(d) $\sqrt{2}$

Ans:- (d)

Q.17. The number of terms in the expansion $\left(-\frac{5}{2x} + \frac{4}{5}\right)^{21}$ is ______.

(a) 20
(b) 21
(c) 22
(d) 23

Ans:- (b)

Q.18. The value of $(n - r)!$, when $n = 11$ and $r = 6$, is ______.

(a) 11!
(b) 6!
(c) 120
(d) 210

Ans:- (c)

Q.19. The number of terms in the expansion $(1 - 2x)^{-\frac{2}{3}}$ is ______.

(a) 6
(b) 9
(c) Infinity
(d) None of these

Ans:- (d)

Q.20. The expansion $(4 - 3x)^{\frac{3}{2}}$ is valid only in terms of power of x if ______.

(a) $|x| < \frac{4}{3}$
(b) $x < \frac{3}{4}$
(c) $x > \frac{3}{4}$
(d) $x = \frac{3}{4}$

Ans:- (a)

Q.21. If ${}^{n}C_7 = {}^{n}C_5$, then n = ______.

(a) 2
(b) 35
(c) 12
(d) None of these

Ans:- (c)

Q.22. $^{2n}C_n$ = ______ .

(a) $\frac{2n!}{(n!)^2}$

(b) $\frac{n!}{2n!}$

(c) $\frac{2n!}{n!}$

(d) $\frac{2(n!)}{(n!)^2}$

Ans:- (a)

Q.23. $^{n}P_2 = 30$, then n = ______.

(a) 3

(b) 4

(c) 5

(d) 6

Ans:- (d)

Q.24. 0! = ______.

(a) 0

(b) 1

(c) 10

(d) None of these

Ans:- (b)

Q.25. $^{n}C_0$ = ______.

(a) 0

(b) 1

(c) n(n-1)

(d) n

Ans:- (b)

Q.26. $^{101}C_{99}$ = ______.

(a) 4040

(b) 3030

(c) 5050

(d) 8080

Ans:- (c)

Q.27. (n-r)! = ______ , when n=7, r = 3.

(a) 42

(b) 24

(c) 8

(d) 84

Ans:- (b)

Q.28. $^{n}C_3 = {}^{n}C_7$, then $^{n}C_8$ = ______.

(a) 42

(b) 43

(c) 44

(d) 45

Ans:- (d)

Q.29. How many different words can be formed with the letters of the word BHARAT?
(a) 360
(b) 630
(c) 180
(d) 315

Ans:- (a)

Q.30. The number of middle term in the expansion of $\left(\frac{a}{2}-\frac{b}{3}\right)^8$ is ______.
(a) 1
(b) 2
(c) 3
(d) None of these

Ans:- (a)

Q.31. The sum of powers of x and y in every term in the expansion of $\left(5x+\frac{2}{y}\right)^{11}$ is ______.
(a) 7
(b) 10
(c) 17
(d) 34

Ans:- (c)

Q.32. The 7^{th} term from the end in the binomial expansion of $\left(2x+\frac{1}{x}\right)^{11}$ is same as ______ from the beginning.
(a) 3^{rd} term
(b) 4^{th} term
(c) 5^{th} term
(d) 6^{th} term

Ans:- (d)

Q.33. Number of middle terms in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$ is ______.
(a) 1
(b) 2
(c) 3
(d) 4

Ans:- (b)

Q.34. Co-efficient of x^5 in $(x+3)^6$ is ______.
(a) 12
(b) 16
(c) 18
(d) 20

Ans:- (c)

Q.35. Find the middle term in the expansion of $\left(x+\frac{1}{x}\right)^4$.
(a) 5
(b) 6
(c) 7

(d) 8

Ans:- (b)

Q.36. If x is so small that its square and higher powers are neglected, then $\frac{1+x}{1-x}=$ _____.

(a) 1+2x

(b) 1-2x

(c) -1-2x

(d) -1 +2x

`

Ans:- (a)

Q.37. The value of $\log_a(1)$ is _______.

(a) 1

(b) 0

(c) ∞

(d) $-\infty$

Ans:- (b)

Q.38. The value of $\log_a(a)$ is _______.

(a) 1

(b) 0

(c) ∞

(d) None of these

Ans:- (a)

Q.39. log(m.n) = _______

(a) logm * logn

(b) $\log(m)^n$

(c) logm +logn

(d) logm – logn

Ans:- (c)

Q.40. $\log (m)^n$ is equal to _______.

(a) m.logn

(b) log(m+n)

(c) log(m-n)

(d) n.logm

Ans:- (d)

Q.41. Logarithmic form of $10^3 = 1000$ is _______.

(a) $3 = \log_{10}1000$

(b) 10 =log3

(c) 30 = log1000

(d) None of these

Ans:- (a)

Q.42. The exponential form of $\log_5 25 = 2$ is _______.

(a) $5^3 = 125$

(b) $2^5 = 32$

(c) $5^2 = 25$

(d) None of these

Ans:- (c)

Q.43. The value of $\log_5 3125$ is _______.

(a) 25
(b) 4
(c) 5
(d) None of these

Ans:- (c)

Q.44. $\log\left(\frac{25}{24}\right)$ is equal to ______.
(a) log(25 - 24)
(b) log25 - log24
(c) log25 + log24
(d) None of these

Ans:- (b)

Q.45. Base change formula of $\log_b m$ is ______.
(a) $\log_m b$
(b) mlogb
(c) $\frac{\log_a m}{\log_b a}$
(d) $\frac{\log_a m}{\log_a b}$

Ans:- (d)

Q.46. Find x, if $\log_x 343 = 3$.
(a) 7
(b) 8
(c) 9
(d) 6

Ans:- (a)

Q.47. If A = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then A is a ______.
(a) Diagonal Matrix
(b) Scalar Matrix
(c) Unit Matrix
(d) None of these

Ans:- (a)

Q.48. If A = $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix}$, then 3A - 4B is equal to ______.
(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 10 \end{bmatrix}$
(b) $\begin{bmatrix} 2 & 1 & 27 \\ 0 & 1 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} 3 & 5 & -5 \\ 0 & -2 & 8 \end{bmatrix}$
(d) None of these

Ans:- (b)

Q.49. If A is a matrix of order 3×5 , then each row of A has ______.
(a) 3 elements
(b) 5 elements
(c) 8 elements

(d) 2 elements

Ans:- (b)

Q.50. If P = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 6 & 8 & 9 \end{bmatrix}$ and Q = $\begin{bmatrix} 2 & 0 & 3 \\ 3 & 0 & 5 \\ 5 & 7 & 0 \end{bmatrix}$, then 2P -3Q is equal to ______.

(a) $\begin{bmatrix} 8 & 4 & 15 \\ 9 & 10 & 29 \\ 27 & 37 & 18 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -4 & 3 \\ 9 & -10 & 1 \\ 3 & 5 & -18 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & 4 & -3 \\ -9 & 10 & -1 \\ -3 & -5 & 8 \end{bmatrix}$

(d) $None\ of\ these$

Ans:- (c)

Q.51. If $\begin{bmatrix} x+3 & 2x \\ 6 & y \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 6 & 3 \end{bmatrix}$, then

(a) x =4 , y = 3

(b) x=3 , y = 4

(c) x = 0 , y = 0

(d) None of these

Ans:- (a)

Q.52. If $\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & 4 \\ 3 & -k \end{bmatrix}$ then k = ______.

(a) 0

(b) 2

(c) -2

(d) 1

Ans:- (b)

Q.53. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then A^2 is equal to ______.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) None of these

Ans:- (c)

Q.54. If A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then AB = ______.

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Ans:- (b)

Q.55. A square matrix A is non-singular if and only if ______.
(a) $A \neq 0$
(b) $|A| = 0$
(c) $|A| \neq 0$
(d) None of these

Ans:- (c)

Q.56. A square matrix is said to be skew- symmetric if and only if ______.
(a) $A^t = A$
(b) $A^t = -A$
(c) $A^t = 0$
(d) None of these

Ans:- (b)

Q.57. The value of $\begin{vmatrix} 5 & -5 \\ 4 & 3 \end{vmatrix}$ is ______.
(a) 5
(b) -35
(c) 35
(d) 15

Ans:- (c)

Q.58. The value of $\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$ is ______.
(a) 1
(b) -1
(c) 0
(d) None of these

Ans:- (a)

Q.59. If $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -5 \\ 4 & 3 \end{vmatrix}$ then value of x is ______.
(a) ± 5
(b) 10
(c) 15
(d) -10

Ans:- (a)

Q.60. The value of $\begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix}$ is ______.
(a) 18
(b) 16
(c) 0
(d) 8

Ans:- (c)

Q.61. Minor of a_{11} in $\begin{vmatrix} 3 & 4 \\ -2 & 7 \end{vmatrix}$ is ______.
(a) 7
(b) -2
(c) 4
(d) 3

Ans:- (a)

Q.62. Minor of a_{21} in $\begin{vmatrix} 3 & 4 \\ -2 & 7 \end{vmatrix}$ is ______.

(a) 3
(b) 4
(c) -2
(d) 7

Ans:- (b)

Q.63. If $\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$, then value of x is ______.

(a) 2
(b) 4
(c) 3
(d) 6

Ans:- (d)

Q.64. The value of $\begin{vmatrix} a & p \\ 0 & b \end{vmatrix}$ by Laplace expansion is ______.

(a) 0
(b) abp
(c) ab
(d) ap

Ans:- (c)

Q.65. In Cramer's rule the system is consistent if ______.

(a) $D = 0, D_1 = 0, D_2 = 0$
(b) $D = 0, D_1 \neq 0, D_2 = 0$
(c) Both (a) and (b) are true
(d) Only (a) is true.

Ans:-(d)

Q.66. In which quadrant the angle 120^0 lies?

(a) I
(b) II
(c) III
(d) IV

Ans:- (b)

Q.67. What is the circular measure of angle 60^0?

(a) π
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$

Ans:- (c)

Q.68. What is the centesimal measure of $\frac{3\pi}{20}$?

(a) 30^g
(b) 20^g
(c) 10^g
(d) 50^g

Ans:- (a)

Q.69. What is the sexagesimal measure of $\frac{2\pi}{3}$?

(a) 90^0

(b) 150^0

(c) 120^0

(d) 360^0

Ans:- (c)

Q.70. 1-cos^2A is equal to ______.

(a) sin^2A

(b) tan^2A

(c) 1- sin^2A

(d) sec^2A

Ans:- (a)

Q.71. sin(90^0-A) and cosA are ______.

(a) Different

(b) Same

(c) Not related

(d) None of the above

Ans:- (b)

Q.72. sec^2A-1 is equal to ______.

(a) cos^2A

(b) tan^2A

(c) sin^2A

(d) cot^2A

Ans:- (b)

Q.73. The value of $\frac{tan60^0}{Cot30^0}$ is equal to______.

(a) 0

(b) 1

(c) 2

(d) 3

Ans:- (b)

Q.74. In ΔABC , right angled at B, AB = 24c.m., BC = 7 c.m., then the value of tan C is ______.

(a) $\frac{12}{7}$

(b) $\frac{24}{7}$

(c) $\frac{20}{7}$

(d) $\frac{7}{24}$

Ans:- (b)

Q.75. (Sin30^0+cos60^0) – (sin60^0+cos30^0) is equal to ______.

(a) 0

(b) 1+$\frac{2}{\sqrt{3}}$

(c) 1 - $\sqrt{3}$

(d) $1 + \sqrt{3}$

Ans:- (c)

Q.76. If x lies in first quadrant and cos x = $\frac{2}{3}$ then tanx is equal to ______.

(a) $\frac{5}{2}$

(b) $\frac{\sqrt{5}}{2}$

(c) $\frac{\sqrt{5}}{3}$

(d) $\frac{2}{\sqrt{5}}$

Ans:- (b)

Q.77. The value of $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$ is equal to ______.

(a) 0

(b) 1

(c) 2

(d) 4

Ans:- (b)

Q.78. sin2A =2sinA is true when A = ______.

(a) 30^0

(b) 45^0

(c) 0^0

(d) 60^0

Ans:- (c)

Q.79. The value of $(\sin 45^0 + \cos 45^0)$ is ______.

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) 1

Ans:- (b)

Q.80. If sinA = $\frac{1}{2}$, then value of cotA is ______.

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$

(d) 1

Ans:- (a)

Q.81. If ΔABC is right angled at C, then the value of cos(A+B) is ______.

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

Ans:- (a)

Q.82. What is the value of $cos^2 20^0 + cos^2 70^0$?

(a) $\sqrt{2}$
(b) 0
(c) 1
(d) None of the above

Ans:- (c)

Q.83. If sinA = $\frac{3}{5}$, then what is the value of cosA ?
(a) $\frac{4}{5}$
(b) $\frac{3}{5}$
(c) $\frac{2}{5}$
(d) $\frac{1}{5}$

Ans:- (a)

Q.84. What is the value of Sin 150^0 ?
(a) $\frac{\sqrt{3}}{2}$
(b) $\frac{1}{2}$
(c) - $\frac{1}{2}$
(d) $-\frac{\sqrt{3}}{2}$

Ans:- (b)

Q.85. The value of cos60^0sin30^0- cos30^0sin60^0 is ______.
(a) $-\frac{1}{2}$
(b) $\frac{1}{2}$
(c) 1
(d) 0

Ans:- (a)

Q.86. The value of cos75^0cos30^0+sin75^0sin30^0 is ______.
(a) $\frac{1}{2}$
(b) - $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) - $\frac{1}{\sqrt{2}}$

Ans:- (c)

Q.87. The value of cos(180^0- θ) is ______.
(a) cosθ
(b) sinθ
(c) –cosθ
(d) –sinθ

Ans:- (c)

Q.88. If A = 60^0 , B = 30^0 then the value of sin(A+B) is ______.
(a) 0
(b) 2

(c) 1
(d) -1

Ans:- (c)

Q.89. The value of $\cos 105^0$ is ______.
(a) $\frac{\sqrt{2}-\sqrt{6}}{4}$
(b) $\frac{\sqrt{2}+\sqrt{6}}{4}$
(c) 0
(d) None of the above

Ans:- (a)

Q.90. The value of sin(-A) is ______.
(a) sinA
(b) –sinA
(c) 0
(d) None of the above

Ans:- (b)

Q.91. The value of sin A and $\cos(90^0 - A)$ are ______.
(a) Same
(b) Different
(c) No relation
(d) Information insufficient

Ans:- (a)

Q.92. If sinA = $\frac{1}{2}$ and cosB = $\frac{1}{2}$, then A+B = ______.
(a) 0^0
(b) 30^0
(c) 60^0
(d) 90^0

Ans:- (d)

Q.93. $\frac{1+tan^2A}{1+cot^2A}$ = ______.
(a) sec^2A
(b) -1
(c) cot^2A
(d) tan^2A

Ans:- (d)

Q.94. $\cot\left(\frac{A}{2}\right) - \tan\left(\frac{A}{2}\right)$ is equal to ______.
(a) tanA
(b) cotA
(c) 2tanA
(d) 2cotA

Ans:- (d)

Q.95. If A = 30^0, then the value of 2 sinA cosA is ______.
(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$
(d) 1

Ans:- (b)

Q.96. In which quadrant the point $(6, -2)$ lies?

(a) 1st (b) 2nd
(c) 3rd (d) 4th

Ans:- (d)

Q.97. The point $(0, -3)$ lies on ______.

(a) x-axis (b) y-axis
(c) Both of these (d) None of these

Ans:- (b)

Q.98. Let (x_1, y_1) be any point, then x_1 is known as ______.

(a) abscissa (b) ordinate
(c) origin (d) None of these

Ans:- (a)

Q.99. Let (x_1, y_1) be any point, then y_1 is known as ______.

(a) abscissa (b) ordinate
(c) origin (d) None of these

Ans:- (b)

Q.100. The point $(0, 0)$ in coordinate plane is known as ______.

(a) abscissa (b) ordinate
(c) origin (d) None of these

Ans:- (c)

Q.101. Let (x, y) represents the Cartesian form and (r, θ) represents the polar form of the point. If $(2, 2)$ is any point, then its polar form is ______.

(a) $(2\sqrt{2},\ 45°)$ (b) $(2, 45°)$
(c) $(\sqrt{2},\ 45°)$ (d) None of these

Ans:- (a)

Q.102. Let (x, y) represents the Cartesian form and (r, θ) represents the polar form of the point. If $(-1, 1)$ is any point, then the value of θ is ______.

(a) $45°$ (b) $-45°$
(c) $135°$ (d) None of these

Ans:- (c)

Q.103. What is the distance between the points $(5, 0)$ and $(0, 0)$?

(a) 0 unit (b) 5 units
(c) 10 units (d) $\sqrt{5}$ units

Ans:- (b)

Q.104. The distance between the points $(0, -4)$ and $(0, -2)$ is ______.

(a) 0 unit (b) 2 units
(c) 4 units (d) None of these

Ans:- (b)

Q.105. The distance of the point $(0, -9)$ from origin is ______.

(a) 9 units (b) 6 units
(c) 3 units (d) None of these

Ans:- (a)

Q.106. If (a, b) and (p, q) be any two points, then mid-point between these points is ______.

(a) $\left(\frac{a-b}{2}, \frac{p-q}{2}\right)$ (b) $\left(\frac{a+b}{n}, \frac{p+q}{n}\right)$
(c) $\left(\frac{a+b}{2}, \frac{p+q}{2}\right)$ (d) None of these

Ans:- (d)

Q.107. If $(-7,4)$ and $(3,-6)$ be any two points, then mid-point between these points is ______.
(a) $(5,-5)$ (b) $(-5,5)$
(c) $(5,5)$ (d) $(-2,-1)$

Ans:- (d)

Q.108. If mid-point between two points is $(3,7)$ and one point between these points is $(4,-2)$, then the second point is ______.
(a) $(16,2)$ (b) $(-2,16)$
(c) $(2,16)$ (d) $(-2,-16)$

Ans:- (c)

Q.109. If mid-point between two points is $(0,0)$ and one point between these points is $(4,-2)$, then the second point is ______.
(a) $(4,-2)$ (b) $(-4,-2)$
(c) $(2,-4)$ (d) $(-4,\ 2)$

Ans:- (d)

Q.110. If vertices of a triangle are $(3,-1)$, $(5,3)$ and $(4,-2)$, then the centroid of the triangle is ______.
(a) $(0,4)$ (b) $(0,0)$
(c) $(4,0)$ (d) $(4,4)$

Ans:- (c)

Q.111. If vertices of a triangle are $(6,a)$, $(-b,5)$ and $(-4,-4)$ with its centroid at origin, then the values of a and b are ______ and ______ respectively.
(a) $-1,-2$ (b) $-2,-1$
(c) $2,-1$ (d) $-1,2$

Ans:- (c)

Q.112. Slope of the straight line, which passes through the points $(-6,3)$ and $(10,-5)$, is ________.
(a) -2 (b) $\frac{1}{2}$
(c) $\frac{16}{8}$ (d) $-\frac{1}{2}$

Ans:- (d)

Q.113. Slope of the straight line, which makes an angle $45°$ with the horizontal, is ________.
(a) -1 (b) 1
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Ans:- (b)

Q.114. If a straight line makes an angle θ with x-axis (horizontal line), then slope of straight line is ______.
(a) $\sin\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\cot\theta$

Ans:- (c)

Q.115. Slope of the straight line $-3x+6y+1=0$ is ________.
(a) -3 (b) -6
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Ans:- (c)

Q.116. The general equation of straight line is ________.

(a) $ax + by + c = 0$ (b) $ax + b = 0$
(c) $y + c = 0$ (d) None of these

Ans:- (a)

Q.117. The straight line parallel to x-axis is ________.

(a) $y = c$ (b) $x = c$
(c) $x + y + c = 0$ (d) None of these

Ans:- (a)

Q.118. Equation of x-axis is ________.

(a) $y = 0$ (b) $x = 0$
(c) $x + y = 0$ (d) None of these

Ans:- (a)

Q.119. Equation of straight line passes through origin with slope m is ________.

(a) $y = mx$ (b) $x = my$
(c) $y = mx + c$ (d) None of these

Ans:- (a)

Q.120. Equation of straight line having slope equal to -1 and y-intercept equal to 4 is ________.

(a) $x + y = 4$ (b) $x + y + 4 = 0$
(c) $4y + x = 0$ (d) None of these

Ans:- (a)

Q.121. Which of the straight line/lines is/are parallel to y-axis?

(a) $x = 1$ (b) $x = 5$
(c) Both of these (d) None of these

Ans:- (b)

Q.122. If a straight line passes through the point $(2, -1)$ and makes an angle $30°$ with x-axis (horizontal line), then equation of straight line is

(a) $x - \sqrt{3}y = 2 + \sqrt{3}$ (b) $x - \sqrt{3}y + 2 + \sqrt{3} = 0$
(c) $y - \sqrt{3}x + 2 + \sqrt{3} = 0$ (d) $y - \sqrt{3}x = 2 + \sqrt{3}$

Ans:- (a)

Q.123. If a straight line cuts of intercepts 4 and 8 on x-axis and y-axis respectively, then equation of straight line is ________.

(a) $\frac{x}{8} + \frac{y}{4} = 1$ (b) $\frac{x}{4} + \frac{y}{8} = 1$
(c) $\frac{x}{4} - \frac{y}{8} = 1$ (d) None of these

Ans:- (b)

Q.124. Which of the following point/points lies/lie on the straight line $x + y = 0$?

(a) $(-1, 1)$ (b) $(5, -5)$
(c) Both of these (d) None of these

Ans:- (c)

Q.125. Which of the following line/lines passes through the origin?

(a) $x - y = 1$ (b) $2x + 2y = 0$
(c) $x + 2y = -1$ (d) None of these

Ans:- (b)

Q.126. Which of the following line passes through the points $(1, 0)$ and $(1, 4)$?

(a) $x = 1$ (b) $y = 0$
(c) $y = 4$ (d) None of these

Ans:- (a)

Q.127. If a straight line passes through the points (x_1, y_1) and (x_2, y_2), then equation of the straight line is______.

(a) $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$ (b) $y + y_1 = \frac{y_2-y_1}{x_2-x_1}(x + x_1)$

(c) $y - y_1 = \frac{y_2+y_1}{x_2+x_1}(x - x_1)$ (d) None of these

Ans:- (a)

Q.128. If α is the inclination of a straight line L passing through the point (x_1 , y_1), then the equation of the straight line in symmetric form is ______.

(a) $y = mx + c$ (b) $y = mx$

(c) $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$ (d) $\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$

Ans:- (d)

Q.129. If the length of perpendicular from the origin to the straight line is 1 and the inclination of this perpendicular to the X-axis is $90°$, then equation of the straight line is ______.

(a) $y = 1$ (b) $x = 1$

(c) $x + y = 1$ (d) None of these

Ans:- (a)

Q.130. Intersecting point of the straight lines $x = 1$ and $x + y = 1$ is ______.

(a) $(1, 2)$ (b) $(-1, -2)$

(c) $(1, 0)$ (d) None of these

Ans:- (c)

Q.131. Intersecting point of the straight lines $y = 0$ and $2x - 3y = 6$ is ______.

(a) $(4, 2)$ (b) $(3, 0)$

(c) $(1, 0)$ (d) None of these

Ans:- (b)

Q.132. Three or more than three straight lines are said to be concurrent if these are intersecting at the same point. The point of intersection of these lines is called point of concurrency. This statement is ______.

(a) Partly True (b) False

(c) True (d) None of these

Ans:- (c)

Q.133. For what value of k, the following lines $x - 2y + 1 = 0$, $2x - 5y + 3 = 0$ and $5x + 9y + k = 0$ are concurrent?

(a) -14 (b) 14

(c) -4 (d) None of these

Ans:- (a)

Q.134. The angle between the straight lines $x = 0$ and $y = 0$ is ______.

(a) $30°$ (b) $45°$

(c) $60°$ (d) $90°$

Ans:- (d)

Q.135. The acute angle between the straight lines, whose slopes are 1 and 0, is ______.

(a) $45°$ (b) $90°$

(c) $60°$ (d) $30°$

Ans:- (a)

Q.136. If m_1 and m_2 are slopes of two straight lines and θ is the acute angle between them, then $\tan\theta =$ ______.

(a) $\left|\frac{m_1+m_2}{m_1-m_2}\right|$ (b) $\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

(c) $\left|\frac{m_1+m_2}{1-m_1m_2}\right|$ (d) None of these

Ans:- (b)

Q.137. If m_1 and m_2 are slopes of two straight lines and $m_1 = m_2$, then the straight lines are______.

(a) perpendicular (b) intersecting

(c) parallel (d) None of these

Ans:- (c)

Q.138. Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be two straight lines and $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the straight lines are ______.

(a) perpendicular (b) intersecting

(c) parallel (d) None of these

Ans:- (c)

Q.139. Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be two straight lines and $a_1\, a_2 + b_1\, b_2 = 0$, then the straight lines are ______.

(a) perpendicular (b) intersecting

(c) parallel (d) None of these

Ans:- (a)

Q.140. The distance from the point $(0\,,\ 0)$ to the straight line $x + 2y + 1 = 0$ is ______.

(a) 1 unit (b) 2 units

(c) $\frac{1}{\sqrt{5}}$ units (d) None of these

Ans:- (c)

Q.141. If (h, k) is centre and r is radius of the circle, then equation of circle is ______.

(a) $x^2 + y^2 = r^2$ (b) $(x-h)^2 + (y-k)^2 = r^2$

(c) $(x+h)^2 + (y+k)^2 = r^2$ (d) None of these

Ans:- (b)

Q.142. If r is radius of the circle centred at origin, then equation of circle is ______.

(a) $x^2 + y^2 = r^2$ (b) $(x-h)^2 + (y-k)^2 = r^2$

(c) $(x+h)^2 + (y+k)^2 = r^2$ (d) None of these

Ans:- (a)

Q.143. If equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, then the centre of circle is ______.

(a) $(g\,, f)$ (b) (f, g)

(c) $(-f, -g)$ (d) $(-g, -f)$

Ans:- (d)

Q.144. If equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, then the radius of the circle is ______.

(a) $\sqrt{g^2 + f^2 - c}$ (b) $\sqrt{g^2 + f^2 + c}$

(c) $\sqrt{g^2 - f^2 - c}$ (d) $\sqrt{g^2 + f^2 - c^2}$

Ans:- (a)

Q.145. General Equation of the circle in xy-plane is ______.

(a) $x + y = 1$ (b) $x^2 + y^2 + 2gx + 2fy + c = 0$

(c) $ax + by + c = 0$ (d) $x^2 + y^2 = 1$

Ans:- (b)

Q.146. If (x_1, y_1) and (x_2, y_2) are end points of a diameter of a circle, then equation of circle is ______.

(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(b) $(x + x_1)(x + x_2) - (y + y_1)(y + y_2) = 0$

(c) $(x + x_1)(x + x_2) + (y + y_1)(y + y_2) = 0$

(d) $(x - x_1)(x - x_2) - (y - y_1)(y - y_2) = 0$

Ans:- (a)

Q.147. MATLAB stands for ______.

(a) Matrix Laboratory (b) Scientific Laboratory

(c) Both of these (d) None of these

Ans:- (a)

Q.148. Which of the following is/are predefined variables/constants in MATLAB?

(a) ans (b) pi

(c) eps (d) All of these

Ans:- (d)

Q.149. What is the default value of i is MATLAB?

(a) 1 (b) -1

(c) $\sqrt{-1}$ (d) None of these

Ans:- (c)

Q.150. MATLAB is a/an ______.

(a) compiler (b) interpreter

(c) Both of these (d) None of these

Ans:- (b)

Very Short Answer Type Questions

Q.1. The angle 60^0 lies in ______ quadrant. (1^{st})

Q.2. In sexagesimal system 1 right angle = ______. (90^0)

Q.3. In centesimal system 1 right angle = ______. (100^g)

Q.4. The value of π in degree = ______. (180^0)

Q.5. 1 radian = ______. $\left(\frac{180^0}{\pi}\right)$

Q.6. The angle 240^0 lies in ______ quadrant. (3^{rd})

Q.7. The circular measure of angle 75^0 is ______. $\left(\frac{5\pi}{12}\right)$

Q.8. The centesimal measure of angle $\frac{2\pi}{3}$ is ______. $\left(\frac{200^g}{3}\right)$

Q.9. The sexagesimal measure of angle $\frac{\pi}{3}$ is ______. (60^0)

Q.10. The value of $sin^2A + cos^2A$ = ______. (1)

Q.11. If sinA = $\frac{2}{3}$, then the value of cosA = ______. $\left(\frac{\sqrt{5}}{3}\right)$

Q.12. If tanA = $\frac{4}{5}$, then the value of secA = ______. $\left(\frac{\sqrt{41}}{5}\right)$

Q.13. The value of sin45^0 = ______. $\left(\frac{1}{\sqrt{2}}\right)$

Q.14. sin(90^0+θ) = ______. (cosθ)

Q.15. tan(360^0-θ) = ______. ($-$tanθ)

Q.16. The value of sin300^0 = ______. $\left(-\frac{\sqrt{3}}{2}\right)$

Q.17. The value of sin28^0cos32^0 + cos28^0sin32^0 = ______. $\left(\frac{\sqrt{3}}{2}\right)$

Q.18. The value of $\frac{tan30^0}{cot30^0}$ = ______. $\left(\frac{1}{3}\right)$

Q.19. Trigonometry helps to study the relationship between ______ and ______ of the triangle. (angles , sides)

Q.20. The angle of ______ is for the objects that are at a level higher than that of the observer. (elevation)

Q.21. The angle of ______ is for objects that are at a level which is lower than that of the observer. (depression)

Q.22. The value of $\frac{tan67^0+tan68^0}{1-tan67^0tan68^0}$ = ______. (-1)

Q.23. The value of cos24^0cos36^0 – sin24^0sin36^0 = ______. $\left(\frac{1}{2}\right)$

Q.24. The angle of elevation of the sun when the length of the shadow of the pole of $\sqrt{3}$ times the height of the pole is ______. (30^0)

Q.25. sin3A = ______. (3sinA – 4sin^3A)

Q.26. Without plotting, find the quadrant in which the point $(-3\,,7)$ lies. (2nd)

Q.27. Without plotting, find the quadrant in which the point $(7,-5)$ lies. (4th)

Q.28. Without plotting, find the quadrant in which the point $(-1\,,-2)$ lies. (3rd)

Q.29. Find the distance between the points $(-4\,,7)$ and $(3\,,8)$. $(\sqrt{50})$

Q.30. Write the distance formula between any two points.

$$\left(\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}\ for\ points\ (x_1,y_1)\ and\ (x_2,y_2)\right)$$

Q.31. Write the formula to find mid-point between any two points.

$$\left(\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\ for\ points\ (x_1,y_1)\ and\ (x_2,y_2)\right)$$

Q.32. Find the mid-point between the points $(10, -7)$ and $(-4, 13)$.
$(3, 3)$

Q.33. Write the formula to find Centroid of a triangle whose vertices are given.
$\left(\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \, for \, vertices \, (x_1, y_1), (x_2, y_2) \, and \, (x_3, y_3)\right)$

Q.34. Find the Centroid of the triangle whose vertices are $(4, -3)$, $(-8, 12)$ and $(-5, 9)$.
$(-3, 6)$

Q.35. Find the distance between the points $(4, -3)$ and $(2, -5)$. $(2\sqrt{2})$

Q.36. Find the mid-point between the points $(5, -3)$ and $(7, 5)$. $(6, 1)$

Q.37. If equation of straight line is $ax + by + c = 0$, then slope of the given straight line is _____.
$\left(-\frac{a}{b}\right)$

Q.38. Find the slope of the straight line which makes an angle $60°$ with X-axis(horizontal).
$(\sqrt{3})$

Q.39. Find the slope of straight line $2x + 6y + 7 = 0$. $\left(-\frac{1}{3}\right)$

Q.40. If a straight line passes through the points (x_1, y_1) and (x_2, y_2), then find the slope of the straight line. $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Q.41. The equation of y-axis is $y = 0$. (TRUE/FALSE) (False)

Q.42. Write the equation parallel of y-axis. (x=k, where k is constant)

Q.43. Write the equation of straight line in Slope-intercept form.
(y=mx+c, where m is slope and c is y-intercept)

Q.44. Write the equation of straight line in Intercept form.
$\left(\frac{x}{a} + \frac{y}{b} = 1, where \, a \, is \, x - intercept \, and \, b \, is \, y - intercept\right)$

Q.45. Does the straight line $2x - y - 1 = 0$ passes through the point $(1, 2)$?
(No)

Q.46. Does the point $(0, -5)$ lies on the straight line $3x - 2y + 1 = 0$?
(No)

Q.47. Give an example of the straight line which passes through the origin.
(x+y=0)

Q.48. Find the intersecting point of the straight lines $x - y = 0$ and $x + y = 2$.
(1, 1)

Q.49. Write the formula to find the angle between two straight lines.
$\left(\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, where \, \theta \, is \, angle \, and \, m_1 \& m_2 \, are \, slopes\right)$

Q.50. Give an example of parallel straight lines. $(x = 1 \,\&\, x = 2)$

Q.51. What is the condition on slopes for perpendicular straight lines (except vertical and horizontal)? $(m_1.m_2 = -1, where \, m_1 \& m_2 \, are \, slopes)$

Q.52. Write the distance formula from point (x_1, y_1) to the straight line $ax + by + c = 0$.

$\left(\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|\right)$

Q.53. If θ is the acute angle between two straight lines, then obtuse angle between them is _____. $(180° - \theta)$

Q.54. Write the standard equation of the circle when its centre and radius are given.

$\left((x-h)^2 + (y-k)^2 = r^2, where\ (h,k)\ is\ centre\ and\ r\ is\ radius\right)$

Q.55. Write the general equation of the circle.

$\left(x^2 + y^2 + 2gx + 2fy + c = 0\right)$

Q.56. Find the equation of the circle having radius 1 and centre at $(0,0)$.

$\left(x^2 + y^2 - 1 = 0\right)$

Q.57. MATLAB is a case sensitive language. (TRUE/FALSE) (TRUE)

Q.58. Scilab stands for Scientific Laboratory. (TRUE/FALSE) (TRUE)

Q.59. The clear command is used just to clear the MATLAB window, not to clear the variable from Workspace. (TRUE/FALSE) (FALSE)

Q.60. Write the syntax to assign the value of 2×3 into the variable x.

$(\gg \boldsymbol{x} = \boldsymbol{2} * \boldsymbol{3})$