

**UNIT - 2**

**MICROWAVE WAVEGUIDES AND COMPONENTS:** Introduction, rectangular waveguides, circular waveguides, microwave cavities, microwave hybrid circuits, directional couplers, circulators and isolators.

**7 Hours**

**TEXT BOOKS:**

- 1.Microwave Devices and circuits-** Liao / Pearson Education.
- 2.Microwave Engineering** – Annapurna Das, Sisir K Das TMH Publication, 2001.

**REFERENCE BOOK:**

- 1. Microwave Engineering** – David M Pozar, John Wiley, 2e, 2004
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## UNIT- 2

## MICROWAVE WAVEGUIDES AND COMPONENTS

**INTRODUCITON**

A waveguide consists of a hollow metallic tube of either rectangular or circular cross section used to guide electromagnetic wave. Rectangular waveguide is most commonly used as waveguide. waveguides are used at frequencies in the microwave range.

At microwave frequencies ( above 1GHz to 100 GHz ) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies . hence microwave signals are propagated through the waveguides in order to minimize the losses.

**Properties and characteristics of waveguide:**

1. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave through multiple reflections .
  2. when the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall .the process results in a component of either electric or magnetic fields in the direction of propagation of the resultant wave.
  3. TEM waves cannot propagate through the waveguide since it requires an axial conductor for axial current flow .
  4. when the wavelength inside the waveguide differs from that outside the guide, the velocity of wave propagation inside the waveguide must also be different from that through free space.
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5. if one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from other end, then there will be complete reflection of the waves resulting in standing waves.

### APPLICATION OF MAXWELLS EQUATIONS TO THE RECTANGULAR WAVEGUIDE:

Let us consider waves propagating along Oz but with restrictions in the  $x$  and/or  $y$  directions. The wave is now no longer necessarily transverse.

The wave equation can be written as

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0 \quad \text{where } k = \frac{\omega}{c}$$

In the present case this becomes

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - k_z^2 + k^2 \right) \vec{H} = 0$$

and similarly for electric field.

There are three kinds of solution possible

TEM	$H_z = E_z = 0$ , i.e. the familiar transverse EM waves
TE	$E_z = 0$
TM	$H_z = 0$

### Boundary conditions:

We assume the guides to be perfect conductors so  $E_t = 0$  inside the guides. Hence, the continuity of  $E_t$  at a boundary implies that  $E_t = 0$  in the wave guide at the boundary.

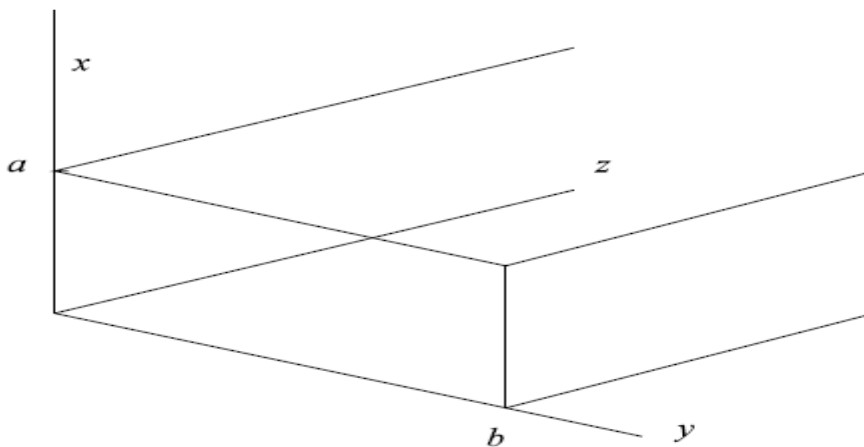
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$E_n$  is not necessarily zero in the wave guide at the boundary as there may be surface charges on the conducting walls (the solution given below implies that there are such charges)

It follows from Maxwell's equation that because  $\nabla \cdot \mathbf{E} = 0$ ,  $E_n$  is also zero inside the conductor (the time dependence of  $\mathbf{E}$  is  $\exp(-i\omega t)$ ). The continuity of  $H_n$  implies that  $H_n = 0$  at the boundary.

There are currents induced in the guides but for perfect conductors these can be only surface currents. Hence, there is no continuity for  $H_t$ . This is to be contrasted with the boundary condition used for waves reflecting off conducting surfaces with finite conductivity.



The standard geometry for a rectangular wave guide is given fig 1. A wave can be guided by two parallel planes for which case we let the planes at  $x = 0, a$  extend to  $y = \pm\infty$ .

**TE Modes:** By definition,  $E_z = 0$  and we start from

$$H_z = H_0 X(x) Y(y) e^{ik_z z}$$

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as the wave equation in Cartesian coordinates permits the use of the separation of variables.

**TM Modes:** By definition,  $H_z = 0$  and we start from

$$E_z = E_0 X(x) Y(y) e^{ik_z z}$$

It is customary in wave guides to use the longitudinal field strength as the reference. For the parallel plate wave guide there is no  $y$  dependence so just set  $Y =$

### TE modes

Using the above form for the solution of the wave equation, the wave equation can be rewritten as

$$\frac{X''}{X} + \frac{Y''}{Y} = k_z^2 - k^2$$

Let  $\frac{X''}{X} = -k_x^2$  and  $\frac{Y''}{Y} = -k_y^2$ ,  $k_x^2 + k_y^2 + k_z^2 = k^2$

the minus signs being chosen so that we get the oscillatory solutions needed to fit the boundary conditions.

Now apply the boundary conditions to determine the restrictions on  $H_z$ .

At  $x = 0, a$ :  $E_y = 0$  and  $H_x = 0$  ( $E_z$  is zero everywhere)

For the following Griffith's writes down all the Maxwell equations specialized to propagation along  $0z$ . I will write just those needed for the specific task and motivate the choice.

We need to relate  $E_y$ ,  $H_x$  to the reference  $H_z$ . Hence, we use the  $y$  component of ME2 (which has 2  $H$  fields and 1  $E$  field)

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega \epsilon_0 E_y$$

The first term is  $ik_z H_x$  which is zero at the boundary.

Consequently,  $\frac{\partial H_z}{\partial x} = 0$  at  $x = 0, a$  and  $X = \cos k_x x$  with

$$k_x = \frac{m\pi}{a}$$

The absence of an arbitrary constant upon integration is justified below.

At  $y = 0, b$ :  $E_x = 0$  and  $H_y = 0$  and we now use the  $x$  component of ME2

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega \epsilon_0 E_x$$

As the second term is proportional  $H_y$  we get

$$\frac{\partial H_z}{\partial y} = 0 \text{ at } y = 0, b \text{ and } Y = \cos k_y y \text{ with } k_y = \frac{n\pi}{b}$$

The general solution is thus

$$\begin{aligned} H_x &= H_0 \cos(k_x x) \cos(k_y y) e^{ik_z z} \\ &= H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{ik_z z} \end{aligned}$$

However,  $m = n = 0$  is not allowed for the following reason.

When  $m = n = 0$ ,  $H_z$  is constant across the waveguide for any  $xy$  plane. Consider the integral version of Faraday's law for a path that lies in such a plane and encircles the wave guide but in the metal walls.

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

As  $E = 0$  in the conducting walls and the time dependence of is given by  $\exp(-i\omega t)$  this equation requires that . We need only evaluate the integral over the guide as  $= 0$  in the walls.

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For constant  $B_z$  this gives  $B_z ab = 0$ . So  $B_z = 0$  as is  $H_z$ . However, as we have chosen  $E_z = 0$  this implies a TEM wave which cannot occur inside a hollow waveguide. Adding an arbitrary constant would give a solution like

$$H_x = H_0 \left[ \cos\left(\frac{m\pi x}{a}\right) + \text{Const} \right] \cos\left(\frac{n\pi y}{b}\right) e^{ik_z z}$$

which is not a solution to the wave equation ... try it. It is also equivalent to adding a solution with either  $m = 0$  or  $n = 0$  which is a solution with a different

### Cut off frequency

This restriction leads to a minimum value for  $k$ . In order to get propagation  $k_z^2 > 0$ . Consequently

$$k^2 > k_x^2 + k_y^2$$

i.e. 
$$\omega^2 > c^2 \pi^2 \left[ \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]$$

Suppose  $a > b$  then the minimum frequency is  $cB/a$  and for a limited range of  $T$  (dependent on  $a$  and  $b$ ) this solution ( $m = 1, n = 0$ , or TE<sub>10</sub>) is the only one possible.

Away from the boundaries

$$ik_z H_x + k_x H_z = -i\omega \epsilon_0 E_y$$

where  $H_{zx}$  means that  $\cos k_x x$  has been replaced by  $\sin k_x x$ .

We need another relation between  $E_y$  and either  $H_x$  or  $H_z$ , which must come from the other Maxwell equation (ME1). We have to decide which component of ME1 to use. If we choose the  $z$  component, the equation involves  $E_x$  and  $E_y$ , introducing another unknown field ( $E_x$ ). However, the  $x$  component involves  $E_y$  and  $E_z$ . As  $E_z = 0$ , this gives the required relation.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\mu_0\omega H_x$$

*i.e.*  $-ik_z E_y = i\mu_0\omega H_x$ , or  $k_z E_y = -\mu_0\omega H_x$

Substituting in the above gives

$$-\frac{ik_z^2 E_y}{\mu_0\omega} + i\omega\epsilon_0 E_y = -k_x H_z^x, \quad E_y = \frac{i\mu_0\omega k_x}{k_x^2 + k_y^2} H_z^x, \text{ etc}$$

$$-k_y H_z^y - ik_z H_y = -i\omega\epsilon_0 E_x$$

and the y component of ME1

$$ik_z E_x = i\mu_0\omega H_y$$

we get

$$-\frac{ik_z^2 E_x}{\mu_0\omega} + i\omega\epsilon_0 E_x = k_y H_z^y, \quad E_x = -\frac{i\mu_0\omega k_y}{k_x^2 + k_y^2} H_z^y$$

### Velocity

The phase velocity  $v_p$  is given by

$$v_p = \frac{\omega}{k_z} = \frac{ck}{k_z} = \frac{ck}{\sqrt{k^2 - k_x^2 - k_y^2}} > c$$

However the group velocity is given by

$$v_g = \frac{\partial\omega}{\partial k_z} = c \frac{\partial k}{\partial k_z} = c \frac{k_z}{k} < c \quad \text{and} \quad v_p v_g = c^2$$

### TM modes

The boundary conditions are easier to apply as it is  $E_z$  itself that is zero at the boundaries.



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Consequently, the solution is readily found to be

$$E_z = E_0 \sin(k_x x) \sin(k_y y) e^{ik_z z}$$

Note that the lowest TM mode is due to the fact that  $E_z \neq 0$ . Otherwise, along with  $H_z = 0$ , the solution is a TEM mode which is forbidden. The details are not given here as the TM wave between parallel plates is an assignment problem.

It can be shown that for ohmic losses in the conducting walls the TM modes are more attenuated than the TE modes.

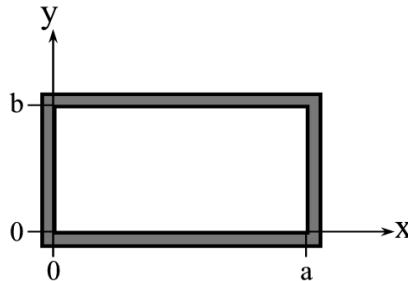
### MAXWELL EQUATIONS

$$\text{ME1} \quad \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{ME2} \quad \vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

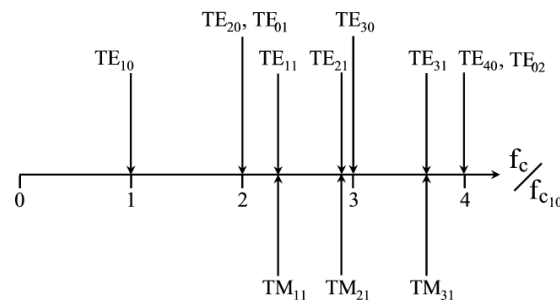
### Rectangular Waveguide:

- Let us consider a rectangular waveguide with interior dimensions are  $a \times b$ ,
- Waveguide can support TE and TM modes.
  - In TE modes, the electric field is transverse to the direction of propagation.
  - In TM modes, the magnetic field that is transverse and an electric field component is in the propagation direction.
- The order of the mode refers to the field configuration in the guide, and is given by  $m$  and  $n$  integer subscripts,  $TE_{mn}$  and  $TM_{mn}$ .
  - The  $m$  subscript corresponds to the number of half-wave variations of the field in the  $x$  direction, and
  - The  $n$  subscript is the number of half-wave variations in the  $y$  direction.
- A particular mode is only supported above its cutoff frequency. The cutoff frequency is given by

## Rectangular Waveguide



## Location of mod



$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\epsilon_o\epsilon_r}} = \frac{1}{\sqrt{\mu_o\epsilon_o}} \frac{1}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

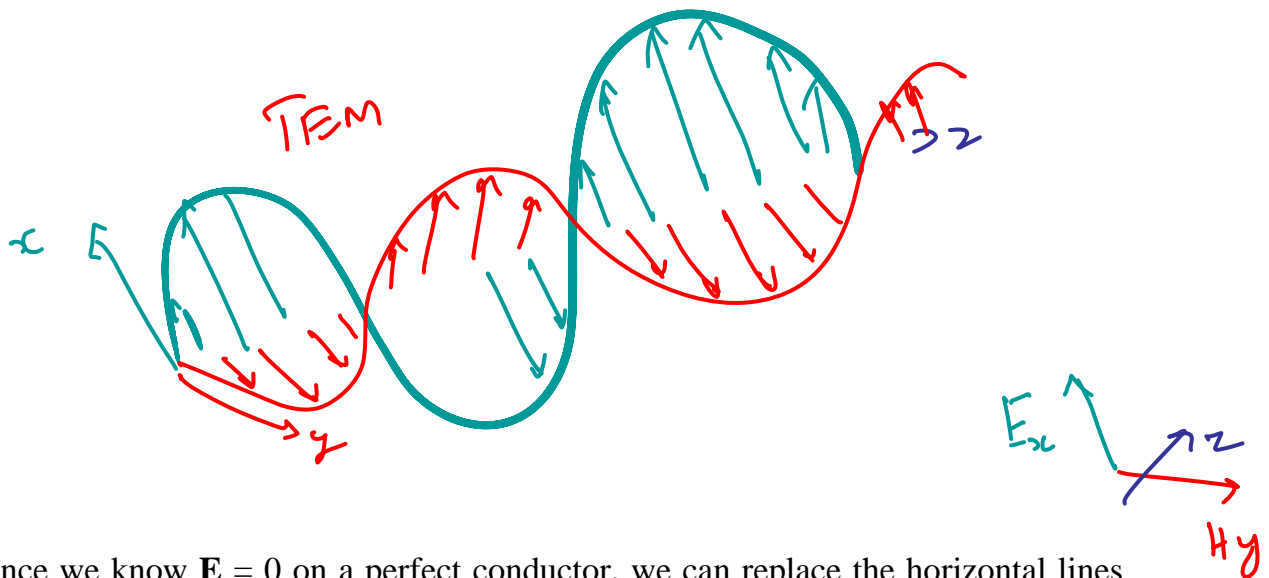
We can achieve a qualitative understanding of wave propagation in waveguide by considering the wave to be a superposition of a pair of TEM waves.

Let us consider a TEM wave propagating in the z direction. Figure shows the wave fronts; bold lines indicating constant phase at the maximum value of the field (+E<sub>o</sub>), and lighter lines indicating constant phase at the minimum value (-E<sub>o</sub>).

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The waves propagate at a velocity  $u$ , where the  $u$  subscript indicates media unbounded by guide walls. In air,  $u = c$ .



Since we know  $\mathbf{E} = 0$  on a perfect conductor, we can replace the horizontal lines of zero field with perfect conducting walls. Now,  $u_+$  and  $u_-$  are reflected off the walls as they propagate along the guide.

The distance separating adjacent zero-field lines in Figure (b), or separating the conducting walls in Figure (a), is given as the dimension  $a$  in Figure (b).

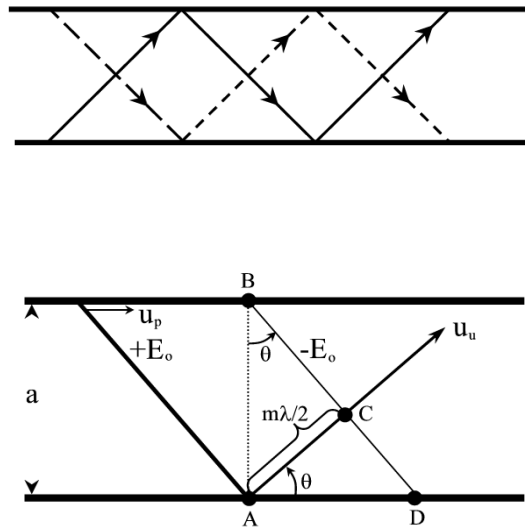
The distance  $a$  is determined by the angle  $\theta$  and by the distance between wavefront peaks, or the wavelength  $\lambda$ . For a given wave velocity  $u$ , the frequency is  $f = u/\lambda$ .

If we fix the wall separation at  $a$ , and change the frequency, we must then also change the angle  $\theta$  if we are to maintain a propagating wave. Figure (b) shows wave fronts for the  $u_+$  wave.

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The edge of a  $+E_o$  wave front (point A) will line up with the edge of a  $-E_o$  front (point B), and the two fronts must be  $\lambda/2$  apart for the  $m = 1$  mode.



For any value of  $m$ , we can write by simple trigonometry

$$\sin \theta = \frac{m \lambda / 2}{a} \quad \Rightarrow \quad \lambda = \frac{2a}{m} \sin \theta = \frac{u_u}{f}$$

The waveguide can support propagation as long as the wavelength is smaller than a critical value,  $\lambda_c$ , that occurs at  $\theta = 90^\circ$ , or

$$\lambda_c = \frac{2a}{m} = \frac{u_u}{f_c}$$

Where  $f_c$  is the cutoff frequency for the propagating mode.

We can relate the angle  $\theta$  to the operating frequency and the cutoff frequency by

$$\sin \theta = \frac{\lambda}{\lambda_c} = \frac{f_c}{f}$$



The *Group velocity* is given by  $u_G = u_u \cos \theta$

The phase constant is given by

$$\beta = \beta_u \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

The guide wavelength is given by

$$\lambda = \frac{\lambda_u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

The ratio of the transverse electric field to the transverse magnetic field for a propagating mode at a particular frequency is the *waveguide impedance*.

For a TE mode, the wave impedance is

$$Z_{mn}^{TE} = \frac{\eta_u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}},$$

For a TM mode, the wave impedance is

$$Z_{mn}^{TM} = \eta_u \sqrt{1 - \left(\frac{f_c}{f}\right)^2}.$$

### General Wave Behaviors:

The wave behavior in a waveguide can be determined by

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Mode	Wave Impedance, $Z$	Guide Wavelength, $\lambda_g$
TEM	$\eta = \sqrt{\frac{\mu}{\epsilon}}$	$\lambda = \frac{1}{f\sqrt{\mu\epsilon}}$
TM	$\eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$\frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$
TE	$\frac{\eta}{\sqrt{1 - (f_c/f)^2}}$	$\frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$

$$H_x^0 = -\frac{1}{h^2} \left( \gamma \frac{\partial H_z^0}{\partial x} - j\omega\epsilon \frac{\partial E_z^0}{\partial y} \right),$$

$$H_y^0 = -\frac{1}{h^2} \left( \gamma \frac{\partial H_z^0}{\partial y} + j\omega\epsilon \frac{\partial E_z^0}{\partial x} \right),$$

$$E_x^0 = -\frac{1}{h^2} \left( \gamma \frac{\partial E_z^0}{\partial x} + j\omega\mu \frac{\partial H_z^0}{\partial y} \right),$$

$$E_y^0 = -\frac{1}{h^2} \left( \gamma \frac{\partial E_z^0}{\partial y} - j\omega\mu \frac{\partial H_z^0}{\partial x} \right),$$

$$H_x^0 = \frac{j\omega\epsilon}{h^2} \frac{\partial E_z^0}{\partial y},$$

$$H_y^0 = -\frac{j\omega\epsilon}{h^2} \frac{\partial E_z^0}{\partial x},$$

$$E_x^0 = -\frac{\gamma}{h^2} \frac{\partial E_z^0}{\partial x},$$

$$E_y^0 = -\frac{\gamma}{h^2} \frac{\partial E_z^0}{\partial y}.$$

- (1) TM mode phase velocity always faster than the light speed in the medium
- (2) TM mode group velocity always slower than the light speed in the medium
- (3) Depends on frequency □ dispersive transmission systems
- (4) Propagation velocity (velocity of energy transport) = group velocity.

Modes of propagation:

Using phasors & assuming waveguide filled with

- lossless dielectric material and
- walls of perfect conductor,

the wave inside should obey...

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 H + k^2 H = 0$$

Then applying on the  $z$ -component where  $k^2 = \omega^2 \mu \epsilon_c$

$$\nabla^2 E_z + k^2 E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0$$

Solving by method of Separation of Variables :

$$E_z(x, y, z) = X(x)Y(y)Z(z)$$

from where we obtain :

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2$$

which results in the expressions :

$$X'' + k_x^2 X = 0$$

$$Y'' + k_y^2 Y = 0$$

$$Z'' - \gamma^2 Z = 0$$

From Faraday and Ampere Laws we can find the remaining four components

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$



Modes of propagation:

From the above equations we can conclude:

- TEM ( $E_z=H_z=0$ ) can't propagate.
- TE ( $E_z=0$ ) transverse electric
  - In TE mode, the electric lines of flux are perpendicular to the axis of the waveguide
- TM ( $H_z=0$ ) transverse magnetic,  $E_z$  exists
  - In TM mode, the magnetic lines of flux are perpendicular to the axis of the waveguide.
- HE hybrid modes in which all components exists.

**TM Mode:**

$$E_z = E_o \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$$

$$H_z = 0$$

$$E_x = -\frac{\gamma}{k^2} \frac{\partial E_z}{\partial x} = -\frac{\gamma}{k^2} \left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

- The  $m$  and  $n$  represent the mode of propagation and indicates the number of variations of the field in the  $x$  and  $y$  directions

### TM Cutoff:

$$\begin{aligned}\gamma &= \sqrt{(k_x^2 + k_y^2) - k^2} \\ &= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}\end{aligned}$$

- The cutoff frequency occurs when

$$\text{When } \omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \text{ then } \gamma = \alpha + j\beta = 0$$

$$\text{or } f_c = \frac{1}{2\pi} \frac{1}{\sqrt{\mu \epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

No propagation, everything is attenuated

$$\text{When } \omega^2 \mu \epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \gamma = \alpha \text{ and } \beta = 0$$

Propagation:

$$\text{When } \omega^2 \mu \epsilon > \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \quad \gamma = j\beta \quad \text{and} \quad \alpha = 0$$

Cutoff

- The cutoff frequency is the frequency below which attenuation occurs and above which propagation takes place. (High Pass)

$$f_{c_{mn}} = \frac{u'}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2}$$

The phase constant becomes

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2} = \beta' \sqrt{1 - \left( \frac{f_c}{f} \right)^2}$$

Phase velocity and impedance

- The phase velocity is defined as

$$u_p = \frac{\omega}{\beta'} \quad \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

- intrinsic impedance of the mode is

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta' \sqrt{1 - \left[ \frac{f_c}{f} \right]^2}$$

### MICROWAVE HYBRID CIRCUITS:

A microwave circuit is formed when several microwave components and devices such as microwave generators, microwave amplifiers, variable attenuators, cavity resonators, microwave filters, directional couplers, isolators

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are coupled together without any mismatch for proper transmission of a microwave signal.

### Scattering matrix :

Let us consider a two port network which represents a number of parameter



$$H \text{ parameters: } \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}$$

$$Y \text{ parameters: } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}$$

$$Z \text{ parameters: } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

$$ABCD \text{ parameters: } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

All the above listed parameters can be represented as the ratio of either voltage to current or current or voltage under certain conditions of input or output ports.

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad (\text{short circuit})$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \quad (\text{open circuit})$$

At microwave frequencies it is impossible to measure :

1. total voltage and current as the required equipment is not available.
2. Over a broad band region, it is difficult to achieve perfect open and short circuit conditions.
3. The active devices used inside the two port network such as microwave power transistors will tend to become unstable under open and short circuit conditions.

### WAVE GUIDE TEE JUNCTIONS:

A waveguide Tee is formed when three waveguides are interconnected in the form of English alphabet T and thus waveguide tee is 3-port junction. The waveguide tees are used to connect a branch or section of waveguide in series or parallel with the main waveguide transmission line either for splitting or combining power in a waveguide system.

There are basically 2 types of tees namely

- 1.) H- plane Tee junction
- 2.) E-plane Tee junction

A combination of these two tee junctions is called a hybrid tee or “ Magic Tee”.

### E-plane Tee(series tee):

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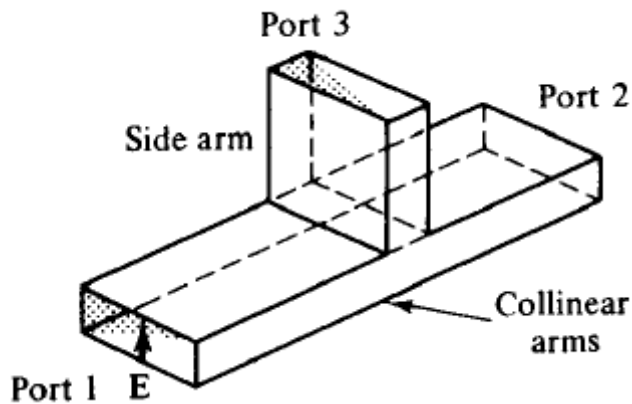
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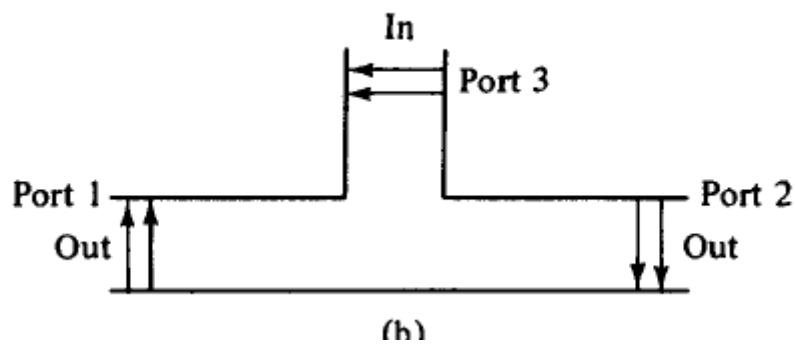
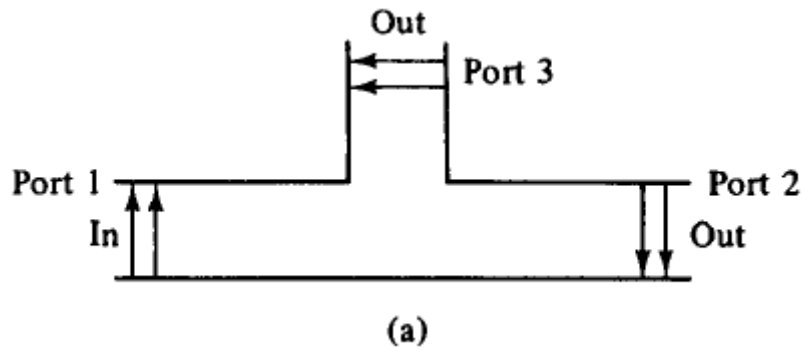
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An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide. If the collinear arms are symmetric about the side arm.

If the E-plane tee is perfectly matched with the aid of screw tuners at the junction, the diagonal components of the scattering matrix are zero because there will be no reflection.

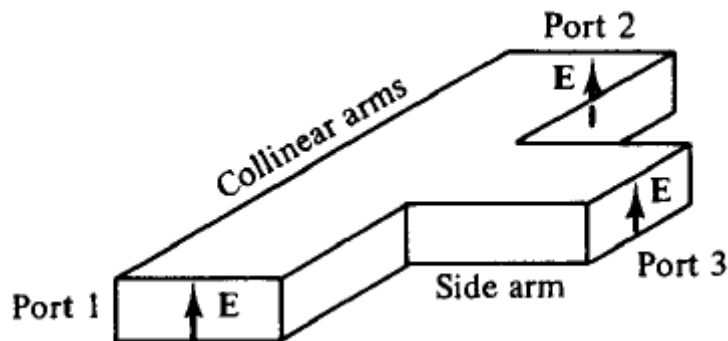
When the waves are fed into side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.





### H-plane tee: (shunt tee)

An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E field or parallel to the H-field of the main guide.

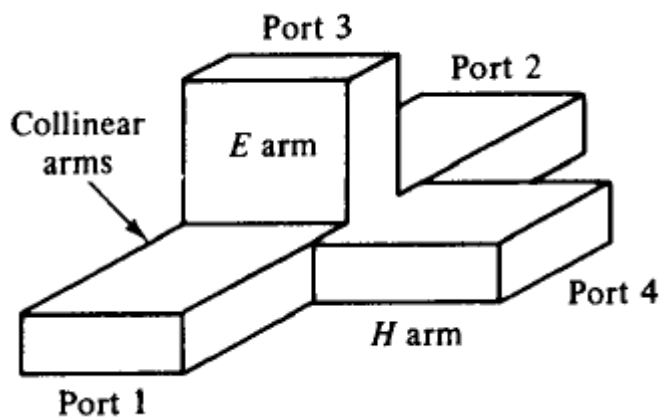


If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive .

If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in same magnitude .

### **Magic Tee ( Hybrid Tees )**

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port 2 the output will be zero at port 3 and additive at port 4.
  2. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.
  3. If a wave is fed into port 3 , it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.
  4. if a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.
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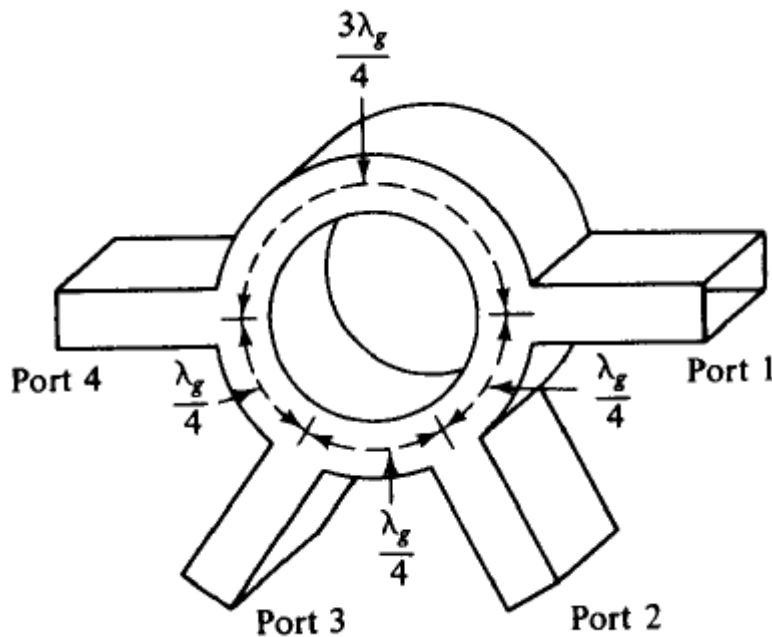


Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

### Hybrid Rings( Rat Race circuits):

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions.



The hybrid ring has characteristics similar to those of the hybrid tee. When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise direction is  $180^\circ$ . Thus the waves are canceled at port 3. For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.

The S matrix for an ideal hybrid ring can be expressed as

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

It should be noted that the phase cancellation occurs only at a designated frequency for an ideal hybrid ring. In actual hybrid rings there are small leakage couplings and therefore the zero elements in the matrix are not equal to zero.

### WAVE GUIDE CORNERS , BENDS AND TWISTS:

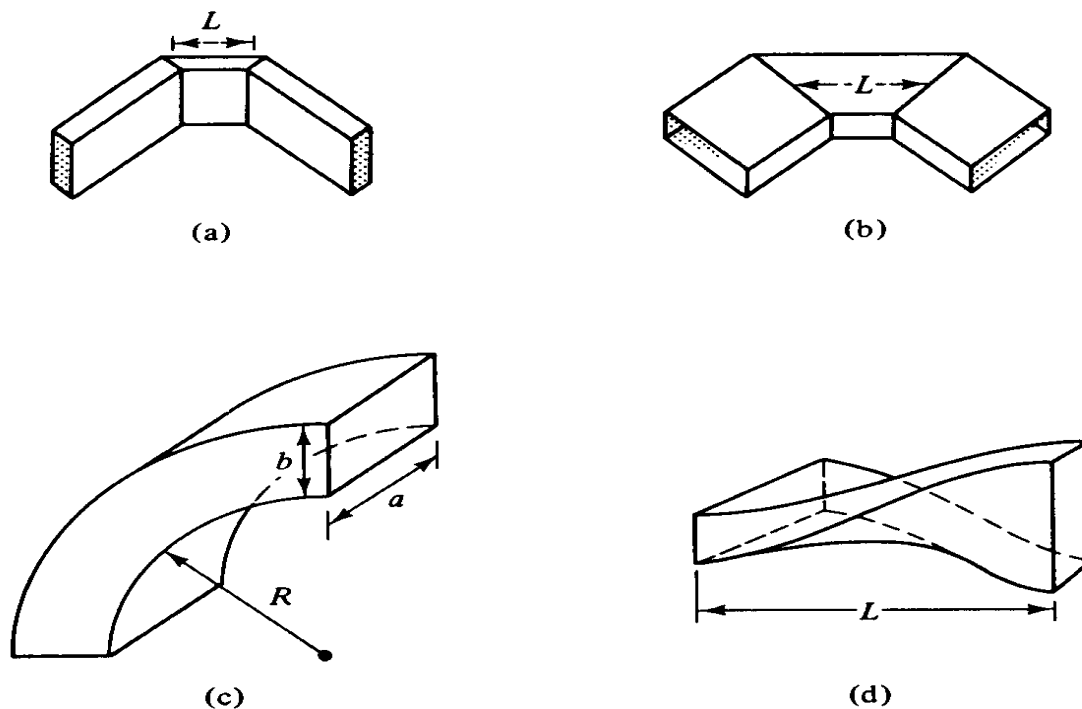
The waveguide corner, bend, and twist are shown in figure below, these waveguide components are normally used to change the direction of the guide through an arbitrary angle.

In order to minimize reflections from the discontinuities, it is desirable to have the mean length  $L$  between continuities equal to an odd number of quarter wave lengths. That is,

$$L = (2n + 1) \frac{\lambda_g}{4}$$

where  $n = 0, 1, 2, 3, \dots$ , and  $\lambda_g$  is the wavelength in the waveguide. If the mean length  $L$  is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled. For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth as

$$R = 1.5b \quad \text{for an } E \text{ bend} \quad R = 1.5a \quad \text{for an } H \text{ bend}$$

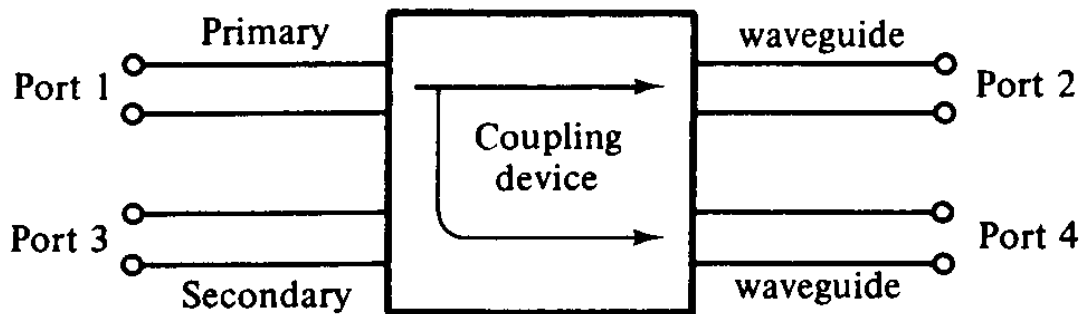


**Waveguide corner, bend, and twist. (a)  $E$ -plane corner. (b)  $H$ -plane corner. (c) Bend. (d) Continuous twist.**

### **DIRECTIONAL COUPLERS:**

A directional coupler is a four-port waveguide junction as shown below. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity. Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined,



Directional coupler.

where  $P_1$  = power input to port 1

$P_3$  = power output from port 3

$P_4$  = power output from port 4

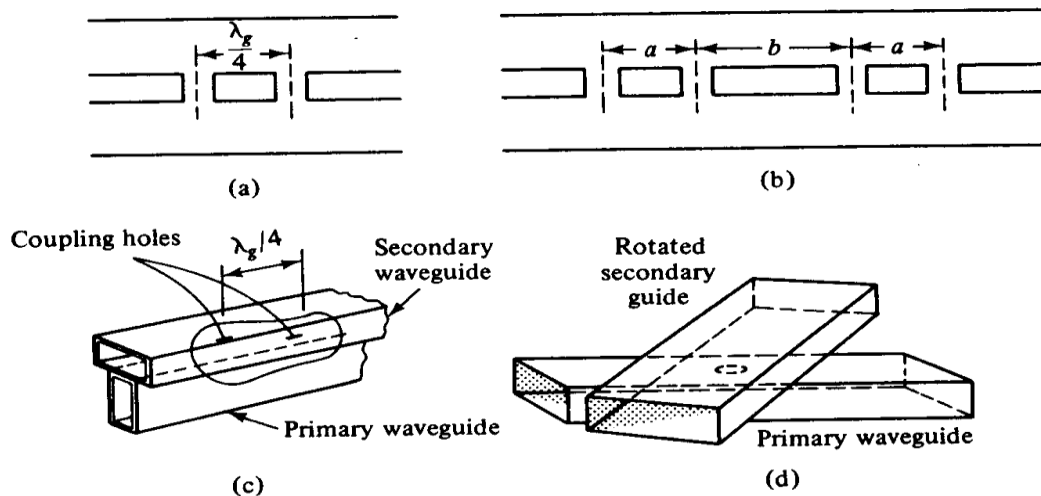
$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1.

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide. An ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched. Actually well-designed directional couplers have a directivity of only 30 to 35 dB.

Several types of directional couplers exist, such as a two-hole direct coupler, four-hole directional coupler, reverse-coupling directional coupler, and Bethe-hole directional coupler. The very commonly used two-hole directional coupler is described here.



**Figure 4-5-2** Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

### TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated. The spacing between the centers of two holes is

$$L = (2n + 1) \frac{\lambda_g}{4}$$

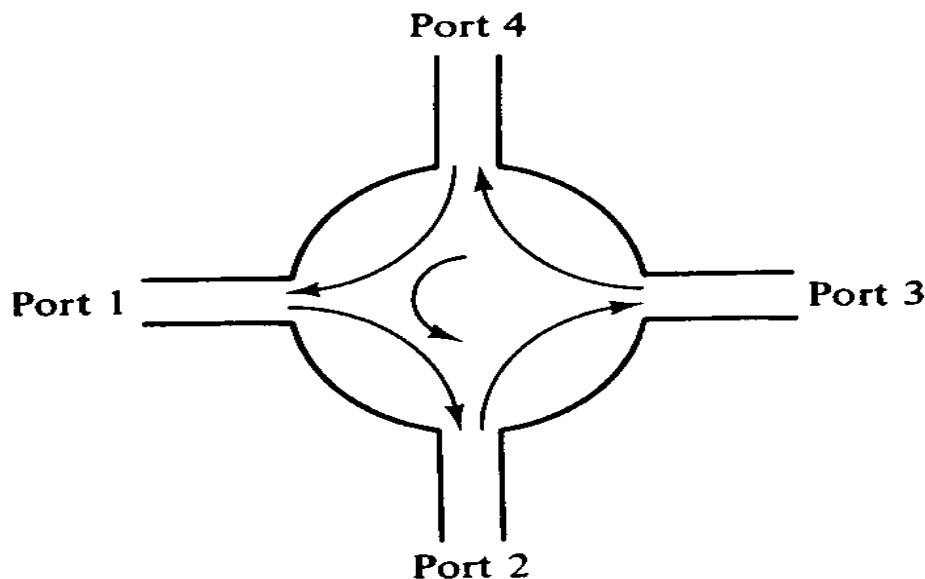
A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas. The forward waves in the secondary guide are in same phase, regardless of the hole space and are added at port 4. The backward waves in the secondary guide are out of phase and are cancelled in port 3.

### CIRCUALTORS AND ISOLATORS:

Both microwave circulators and isolators are non reciprocal transmission devices that use the property of Faraday rotation in the ferrite material. A non reciprocal phase shifter consists of thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized. When a piece of ferrite is affected by a dc magnetic field the ferrite exhibits Faraday rotation. It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor.

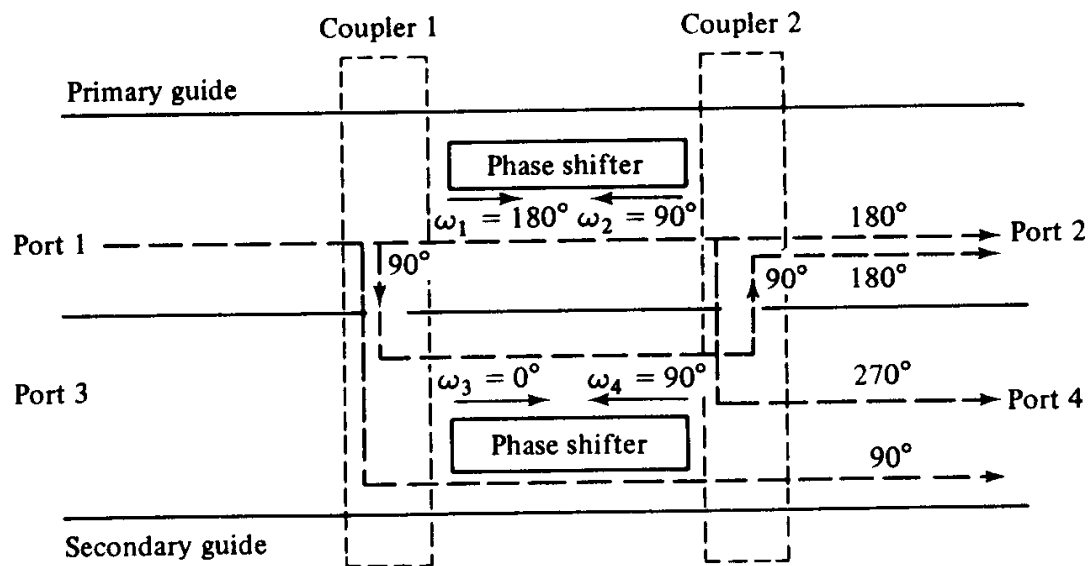
### **MICROWAVE CIRCULATORS:**

A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the  $n$ th port to the  $(n + 1)$ th port in one direction. Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.



**The symbol of a circulator.**

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## Schematic diagram of four-port circulator.

The operating principle of a typical microwave circulator can be analyzed with the aid of Fig shown above. Each of the two 3-dB couplers in the circulator introduces a phase shift of  $90^\circ$ , and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1, the wave is split into two components by coupler I. The wave in the primary guide arrives at port 2 with a relative phase' change of  $180^\circ$ . The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of  $180^\circ$ . Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of  $270^\circ$ . The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of  $90^\circ$ . Since the two waves reaching port 4 are out of phase by  $180^\circ$ , the power transmission from port 1 to port 4 is zero. In general, the differential

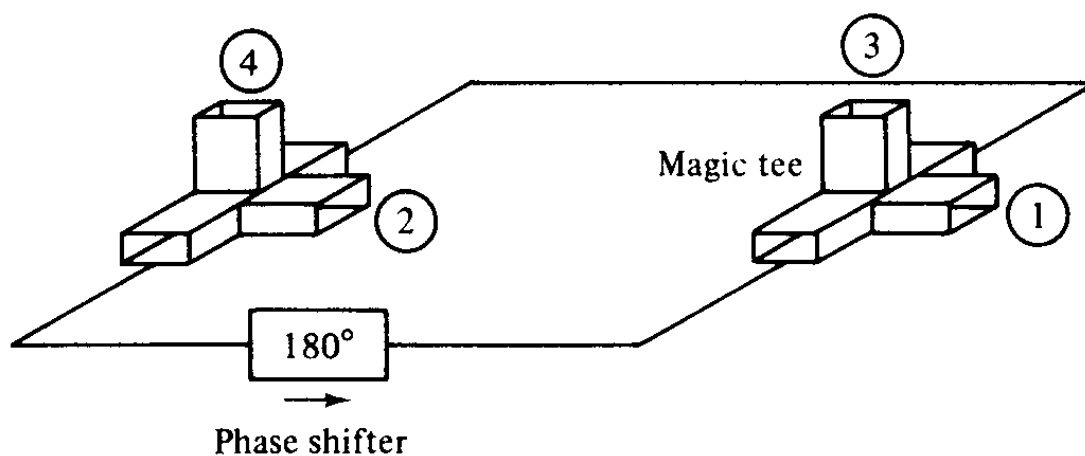
propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \quad \text{rad/s}$$

$$\omega_2 - \omega_4 = 2n\pi \quad \text{rad/s}$$

where  $m$  and  $n$  are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as  $1 \sim 2 \sim 3 \sim 4 \sim 1$ .

Many types of microwave circulators are in use today. However, their principles of operation remain the same. A four-port circulator is constructed by the use of two magic tees and a phase shifter. The phase shifter produces a phase shift of  $180^\circ$ .



**A four-port circulator.**



A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S parameters the S-matrix is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

### MICROWAVE ISOLATORS:

An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called *uniline*.

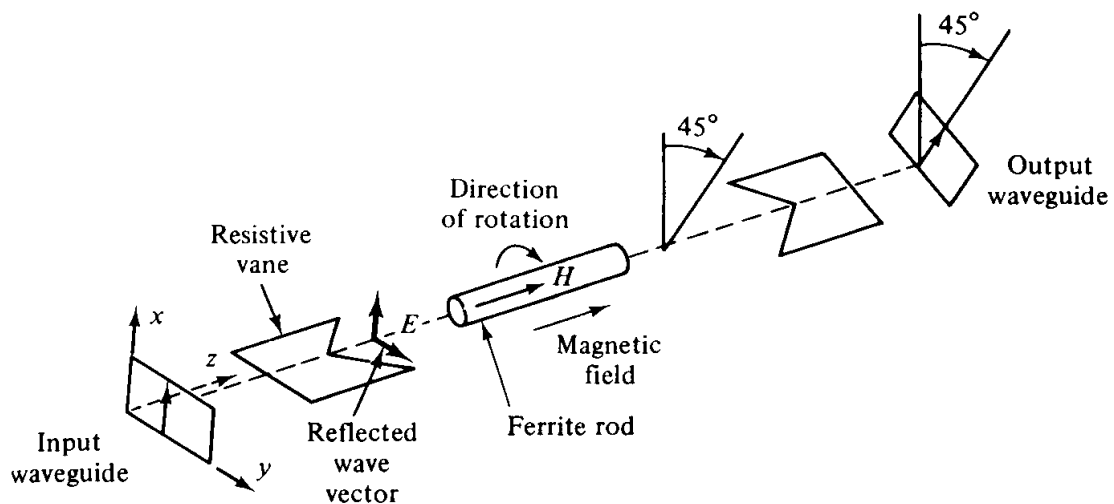
Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown below.

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The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows. The input resistive card is in the  $y$ - $z$  plane, and the output resistive card is displaced  $45^\circ$  with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by  $45^\circ$ . The degrees of rotation depend on the length and diameter of the rod and on the applied dc magnetic field. An input TE<sub>10</sub> dominant mode is incident to the left end of the isolator. Since the TE<sub>10</sub> mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation. The wave in the ferrite rod section is rotated clockwise by  $45^\circ$  and is normal to the output resistive card. As a result of rotation, the wave arrives at the output.



end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise  $45^\circ$  by the ferrite rod. However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation.

### **RECOMMENDED QUESTIONS ON UNIT – 2**

1. Discuss the various properties and characteristics of waveguides.
  2. Show that waveguide acts as a high pass filter
  3. Derive expressions for cutoff wavelength and cutoff frequency for TM waves propagating through rectangular waveguides.
  4. Derive expressions for guide wavelength , phase and group velocity for TM waves in RWG
  5. Draw the field patterns for the dominant TM and TE modes in rectangular waveguides.
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6. Discuss the various types of losses occurring in rectangular waveguides.
7. Obtain an expression for attenuation in co-axial lines.
8. Derive an expression for frequency of oscillation for a rectangular and cylindrical resonator.
9. List the applications of cavity resonators.
10. Draw a neat diagram of H-plane Tee and explain its operation and derive the S matrix.
11. Draw a neat diagram of E-plane Tee and explain its operation and derive the S matrix.
12. Draw a neat diagram of Magic Tee and explain its operation and derive the S matrix.
13. Explain the 2 hole directional coupler with sketch.
14. Explain the operation of a 3 port circulator
15. Explain the working of faraday rotation isolator.

### UNIT - 3

#### MICROWAVE DIODES,

Transfer electron devices: Introduction, GUNN effect diodes – GaAs diode, RWH theory, Modes of operation, Avalanche transit time devices: READ diode, IMPATT diode, BARITT diode, Parametric amplifiers ,Other diodes: PIN diodes, Schottky barrier diodes.

**7 Hours**

#### TEXT BOOKS:

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## Microwaves and Radar

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1. **Microwave Devices and circuits-** Liao / Pearson Education.
2. **Microwave Engineering** – Annapurna Das, Sisir K Das TMH Publication, 2001.

### REFERENCE BOOK:

1. **Microwave Engineering** – David M Pozar, John Wiley, 2e, 2004
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